A-Level

Physics
Xavier International College

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Chapter- 15 Stationary Waves

Key Points

1 From Principle of superposition of waves

2 Nodes and antinodes

3 Formation of stationary waves

4 Determining the wavelengths and speed of sound

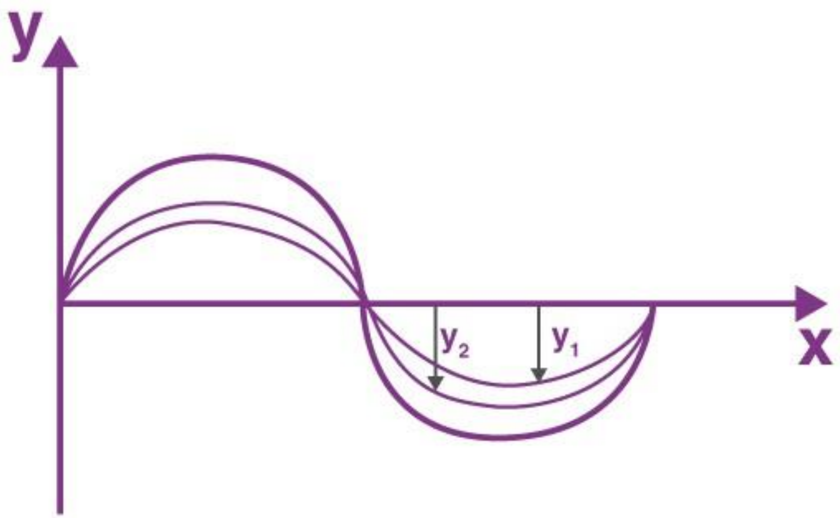
Superposition Principle

- The principle of superposition states that when two or more waves with the same frequency travelling in opposite directions overlap, the resultant displacement is the sum of displacements of each wave
- This principle describes how waves which meet at a point in space interact

if $y_1, y_2, y_3, y_4, y_5,$ ……… are the displacement of individual waves than displacement of resultant wave is,

$$y_n = y_1 + y_2 + y_3 + y_4, \dots\dots$$

- It is possible for all kind of waves like light, sound and string waves.



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Stationary Waves

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- Stationary waves, or standing waves, are produced by the superposition of two waves of the same frequency and amplitude travelling in opposite directions.
- This is usually achieved by a travelling wave and its reflection. The superposition produces a wave pattern where the peaks and troughs do not move.
- Like progressive waves stationary waves are may be transverse or longitudinal wave.
- The transverse stationary waves are formed in vibrating strings of guitar, piano and The longitudinal stationary wave are formed in organ pipes and in the air column of resonance tube.
- There is no flow of energy along the wave.

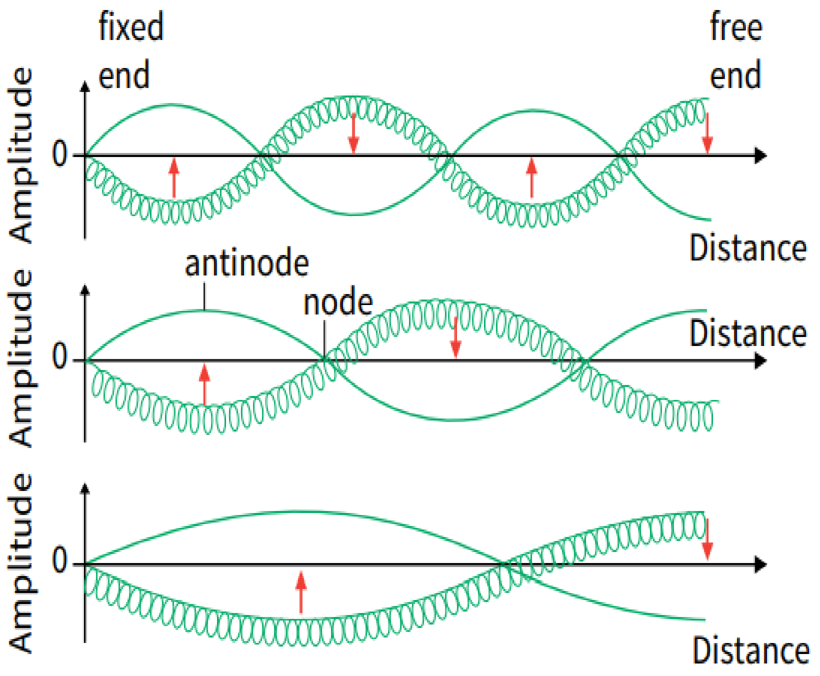


Figure 15.3 Different stationary wave patterns are possible, depending on the frequency of vibration.

Nodes and Antinodes

- In the stationary wave, there are some points that are permanently at rest i.e. they have zero displacement is called nodes.
- There are another set of points between nodes which have the maximum displacement or vibration is called antinodes.

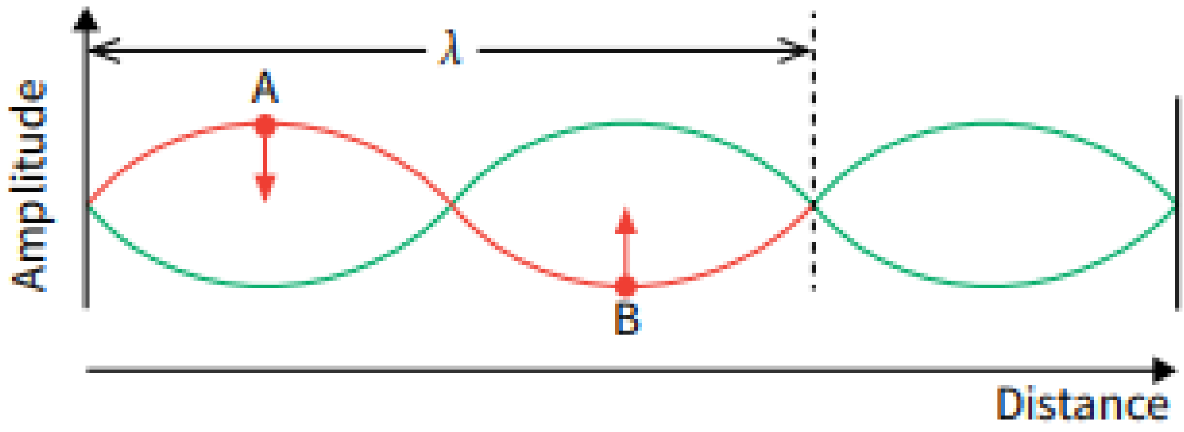


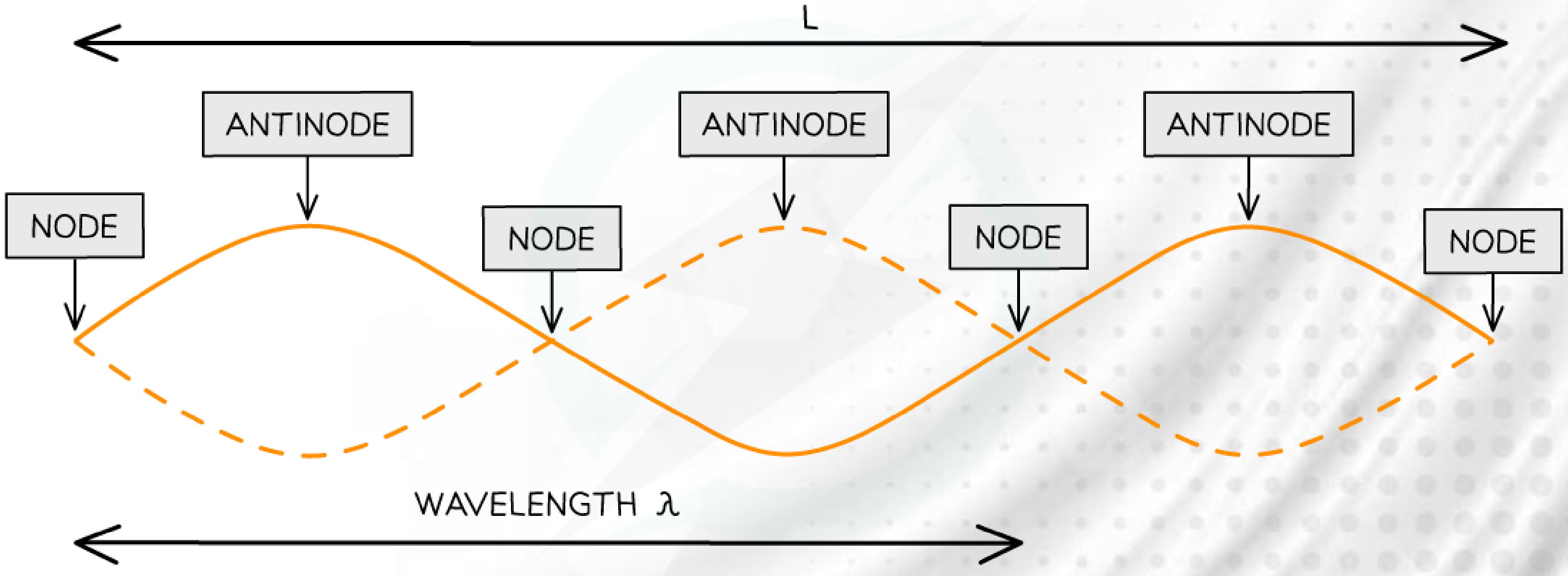
Figure 15.4 The fixed ends of a long spring must be nodes in the stationary wave pattern.

Formation of Stationary Waves

- A stationary wave is made up of nodes and antinodes

  Nodes are where there is no vibration

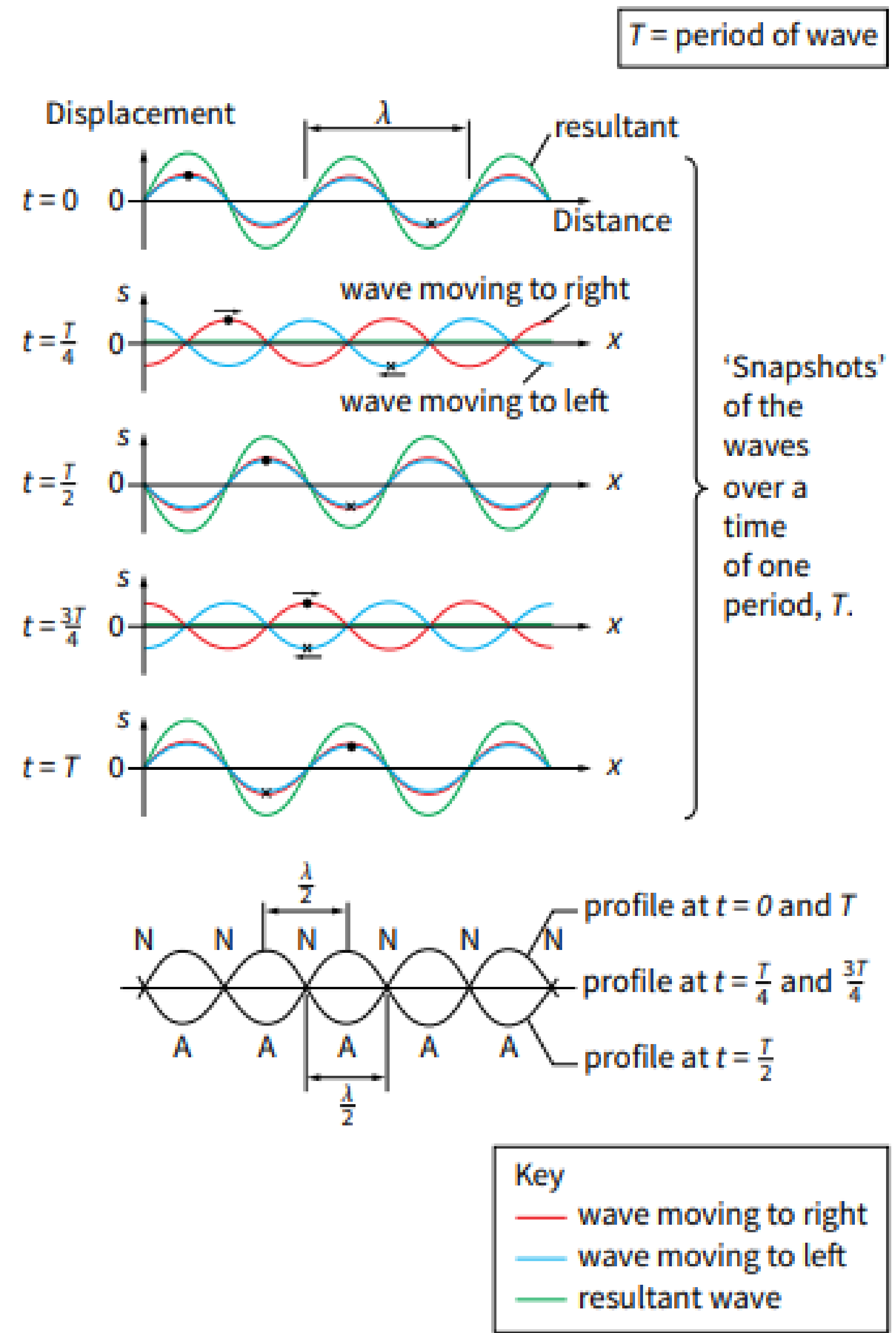
  Anti nodes are where the vibrations are at their maximum amplitude.
- The nodes and antinodes donot move along the string. Nodes are fixed and antinodes only move in the vertical direction.



- Between nodes, all points on the stationary are in phase.
- *L* is the length of the string
- 1 wavelength *λ* is only a portion of the length of the string

Illustrate the formation of stationary wave along a long spring

- At time t = 0, the progressive waves travelling to the left and right are in phase. The waves combine constructively, giving an amplitude twice that of each wave.
- After a time equal to one-quarter of a period (t = T/ 4), each wave has travelled a distance of one quarter of a wavelength to the left or right. Consequently, the two waves are in antiphase (phase difference = 180°). The waves combine destructively, giving zero displacement
- After a time equal to one-half of a period (t = T /2), the two waves are back in phase again. They once again combine constructively.
- After a time equal to three-quarters of a period (t = 3T 4), the waves are in antiphase again. They combine destructively, with the resultant wave showing zero displacement.
- After a time equal to one whole period (t = T), the waves combine constructively


T = period of wave
Displacement
λ
resultant
t = 0
0
Distance
s
t = T/4
wave moving to right
wave moving to left
x
t = T/2
t = 3T/4
t = T
'Snapshots' of the waves over a time of one period, T.
λ/2
N
A
profile at t = 0 and T
profile at t = T/4 and 3T/4
profile at t = T/2
Key
wave moving to right
wave moving to left
resultant wave

- The separation between adjacent nodes and antinodes is related to the wavelength λ of the progressive wave.

  Separation between two adjacent nodes or between two adjacent antinodes $=\frac{\lambda}{2}$

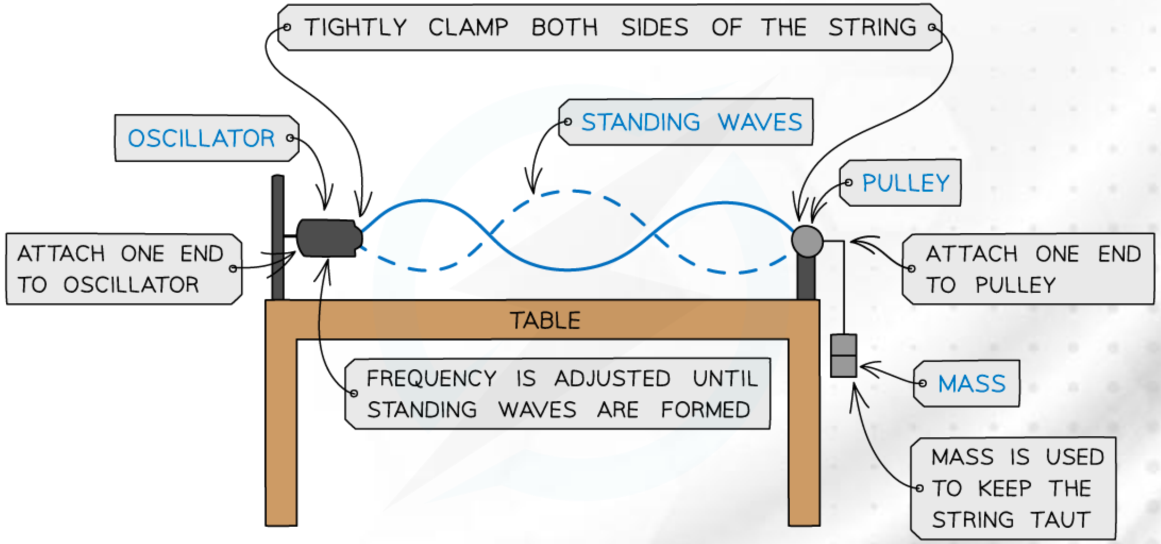
  Separation between adjacent node and antinodes $= \frac{\lambda}{4}$

- Wavelength of progressive wave is used to determine the speed and frequency f by using wave equation but for stationary wave speed is zero

  i.e. $v = f\lambda$

Stretched strings

- Vibrations caused by stationary waves on a stretched string produce sound. This is how stringed instruments, such as guitars or violins, work
- This can be demonstrated by a length of string under tension fixed at one end and vibrations made by an oscillator:
- As the frequency of the oscillator changes, standing waves with different numbers of minima (nodes) and maxima (antinodes)



Microwaves

- A microwave source is placed in line with a reflecting plat and a small detector between the two
- The reflector can be moved to and from the source to vary th stationary wave pattern formed.
- By moving the detector, it can pick up the minima (nodes) an maxima (antinodes) of the stationary wave pattern
- If the probe is moved along the direct line from the transmitte to the plate, the wave of the microwave can be determine from the distance between the nodes.
- And frequency of microwave is c= f λ (c= speed of light)

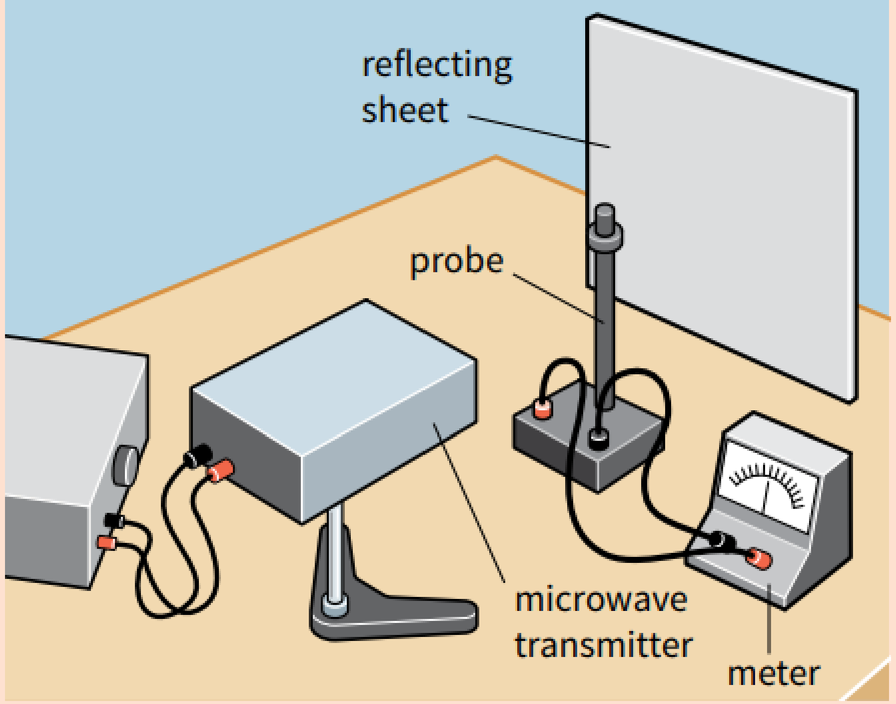
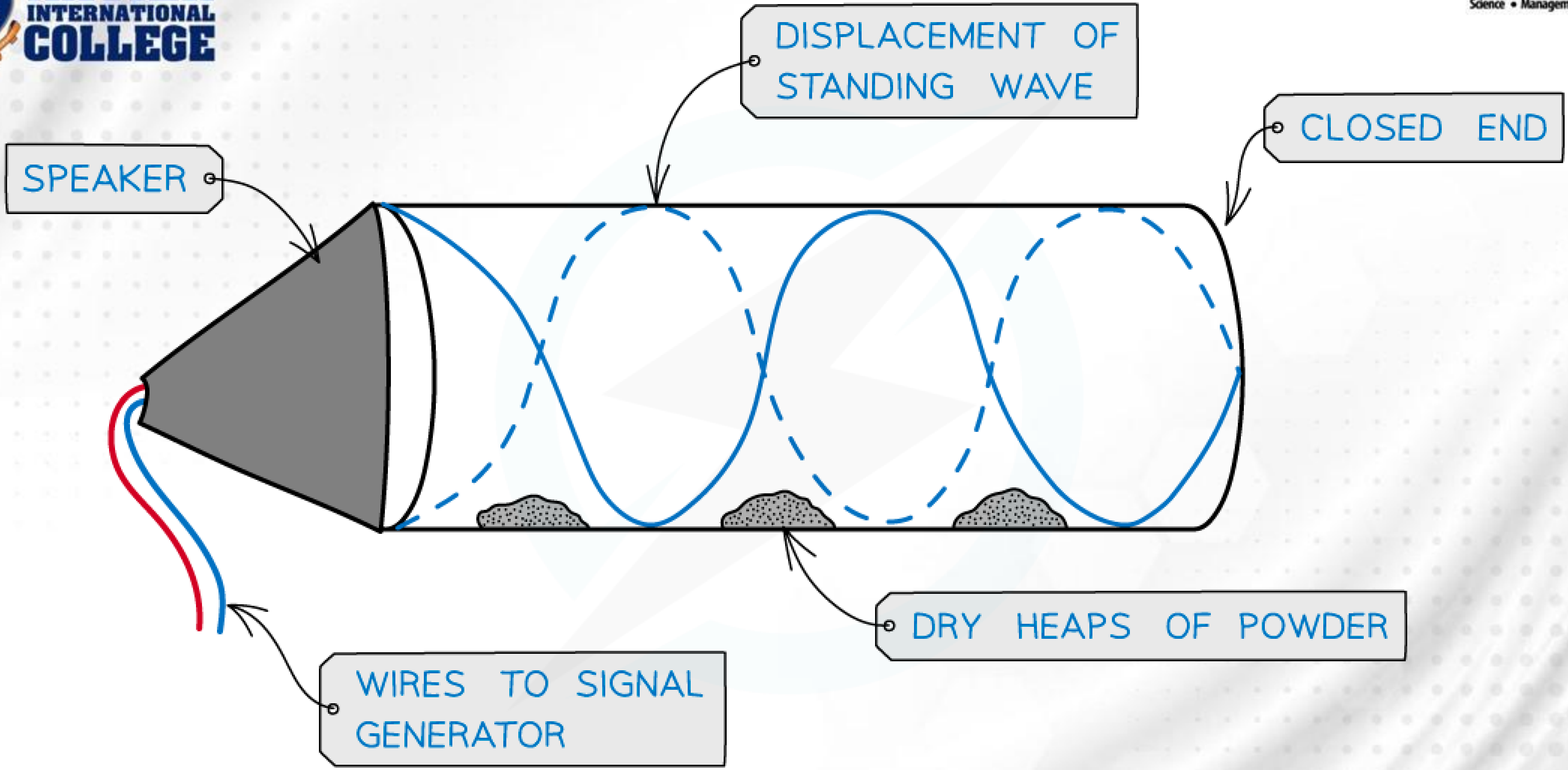


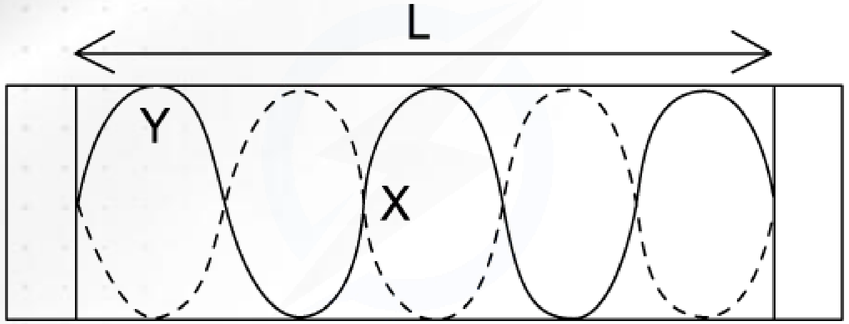
Figure 15.8 A stationary wave is created when microwaves are reflected from the metal sheet.

Air Columns

- The formation of stationary waves inside an air column can be produced by sound waves This is how musical instruments , such as clarinets and organs pipe work.
- This can be demonstrated by placing a fine powder inside the air column and a loudspeaker at the open end.
- At certain frequencies, the powder forms evenly spaced heaps along the tube, showing where there is zero disturbance as a result of the nodes of the stationary wave
- In order to produce a stationary wave, there must be a minima (node) at one end and a maxima (antinode) at the end with the loudspeaker
- The maximum vibration of sound in the air when external force is applied with same frequency is called Resonance frequency.


DISPLACEMENT OF STANDING WAVE
CLOSED END
SPEAKER
DRY HEAPS OF POWDER
WIRES TO SIGNAL GENERATOR

1. A stretched string is used to demonstrate a stationary wave, as shown in the diagram.



Which row in the table correctly describes the length of L and the name of **X** and **Y**?

	Length L	Point X	Point Y
A	5 wavelengths	Node	Antinode
B	$2\frac{1}{2}$ wavelengths	Antinode	Node
C	$2\frac{1}{2}$ wavelengths	Node	Antinode
D	5 wavelengths	Antinode	Node

- A hollow wooden or metallic tube used to produce musical sound is called an organ pipe.
- The air column in an organ pipe is set into vibrations by blowing air into it form one end.

Harmonics and overtones

- Harmonics represents the frequencies of the overtones are integral multiples of the frequency of the fundamental.
- The fundamental frequency f is called the first harmonic and 2f is known as the second harmonic.
- Overtones is a term applied to any higher frequency standing wave.
- An overtone is defined as any frequency produced by an instrument which is greater than fundamental frequency and also define as overlapping of harmonics.
- First overtone is called second harmonics and so on.

Measuring Wavelength

- Stationary waves have different wave patterns depending on the frequency of the vibration and the situation in which they are created.

Two fixed ends

- When a stationary wave, such as a vibrating string, is fixed at both ends, the simplest wave pattern is a single loop made up of two nodes and an antinode
- This is called the **fundamental mode** of vibration or the **first harmonic**
- The particular frequencies (i.e. resonant frequencies) of standing waves possible in the string depend on its length L and its speed v
- As you increase the frequency, the higher harmonics begin to appear
- The frequencies can be calculated from the string length and wave equation
- The nth harmonics has n antinodes and n+1 nodes.

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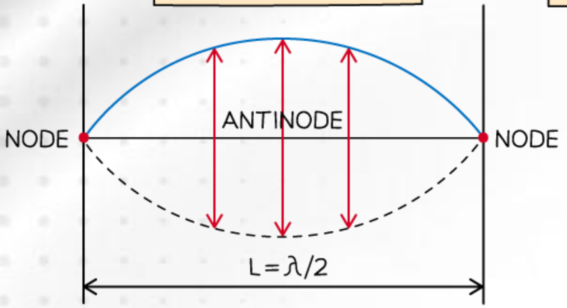
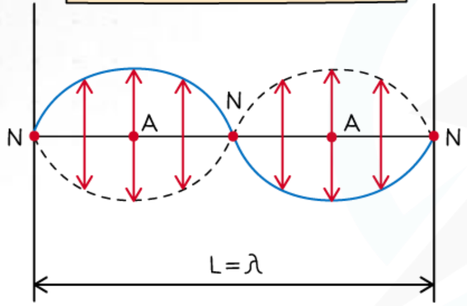
FUNDAMENTAL	WAVELENGTH λ	FREQUENCY $= \frac{c}{\lambda}$



$\lambda = 2L$

$f = \frac{c}{2L}$

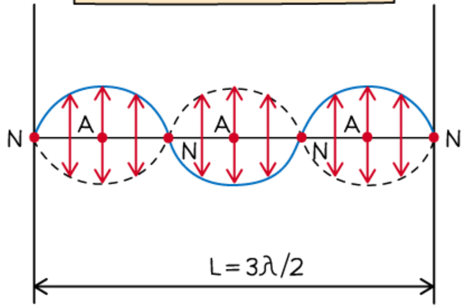
SECOND HARMONIC



$\lambda = L$

$f = \frac{c}{L}$

THIRD HARMONIC



$\lambda = \frac{2L}{3}$

$f = \frac{3c}{2L}$

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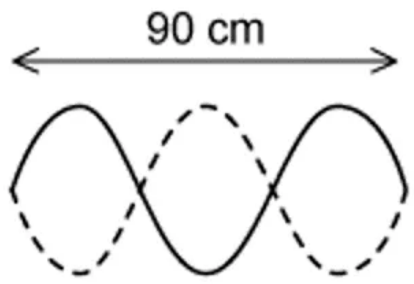
A stationary wave shown in the diagram was created on a stretched string, it had two instants of maximum vertical displacement.



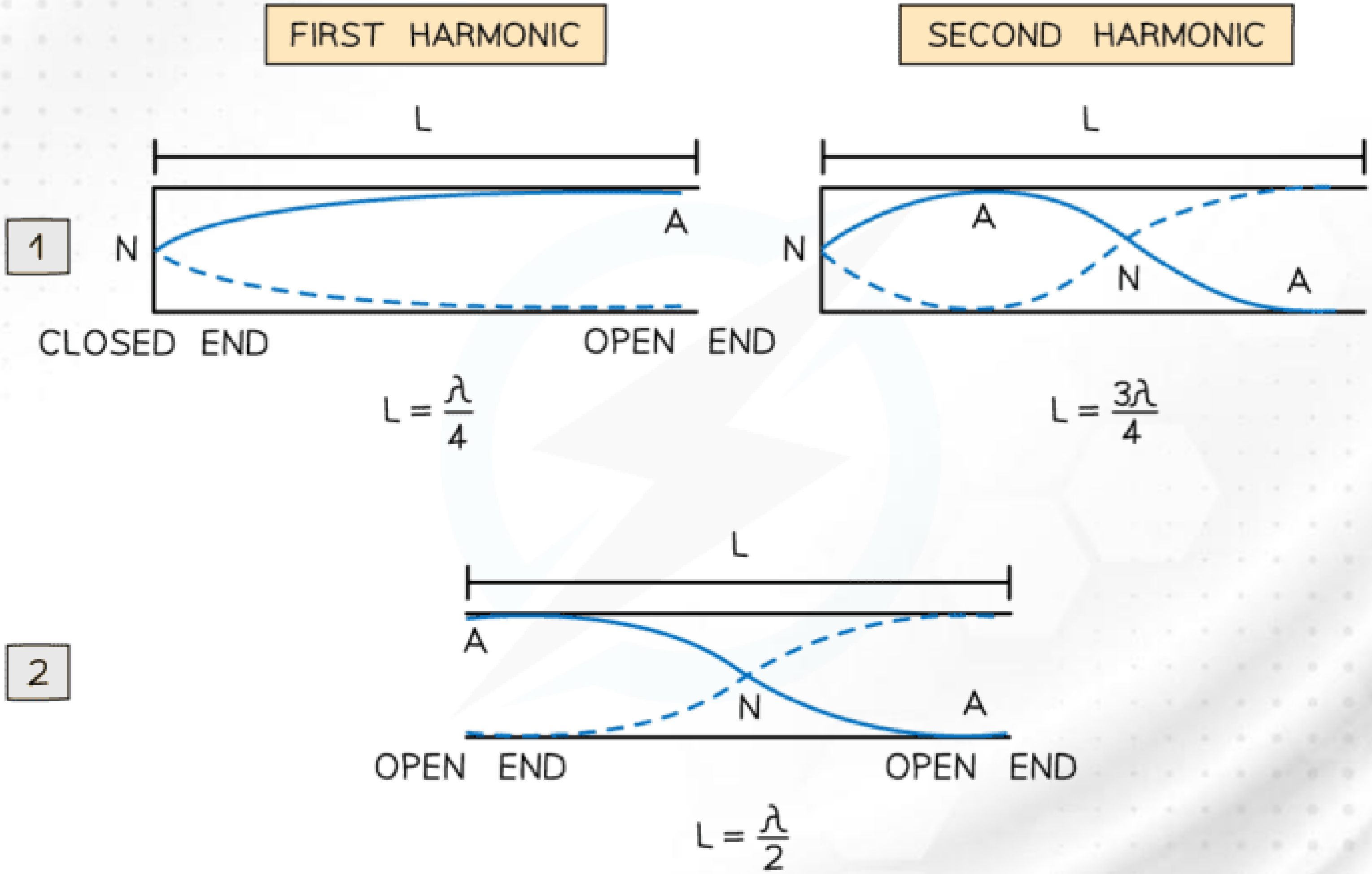
The frequency of the wave is 13 Hz.

What is the speed of the wave?

A 3.9 m s^{-1} **B** 7.8 m s^{-1} **C** 390 m s^{-1} **D** 780 m s^{-1}

One or two open ends in air column

- When a stationary wave is formed in an air column with one or two open ends we see slightly different wave patterns in each.



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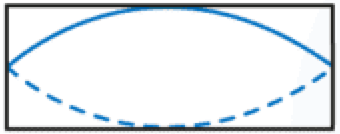
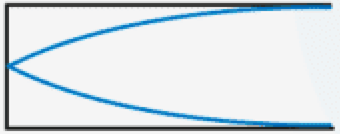
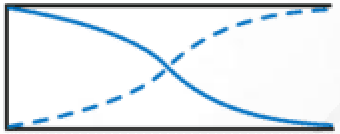
- In Image 1: only one end of the air column is open, so, the fundamental mode is now made up of a quarter of a wavelength with one node and one antinode
  Every harmonic after that adds on an extra node or antinode
- In Image 2: the column is open on both ends, so, the fundamental mode is made up one node and two antinodes
- In summary, a column length L for a wave with wavelength λ and resonant frequency f for stationary waves to appear is as follows

Air column fundamental wave	Length L / m	Resonant frequencies f / Hz	Value of n
	$L = \frac{n\lambda}{2}$	$f = \frac{nv}{2L}$	$n = 1, 2, 3$
	$L = \frac{n\lambda}{4}$	$f = \frac{nv}{4L}$	$n = \text{odd}$
	$L = \frac{n\lambda}{2}$	$f = \frac{nv}{2L}$	$n = 1, 2, 3...$

Stationary waves and musical instruments

- The production of different notes by musical instruments often depends on the creation of stationary waves.
- For a guitar, the two ends of a strings are fixed so nodes must be established at those points.

when the string is plucked half way along its length, it vibrates with an antinode at its midpoint. This is known as the fundamental mode of vibration.

- The sounds that are produced are made up of several different stationary waves having different patterns of nodes and antinodes.

Determining the wavelength and speed of sound

Since we know that adjacent nodes (or antinodes) of a stationary wave are separated by half a wavelength, we can use this fact to determine the wavelength λ of a progressive wave. If we also know the frequency f of the waves, we can find their speed v using the wave equation $v = f\lambda$.

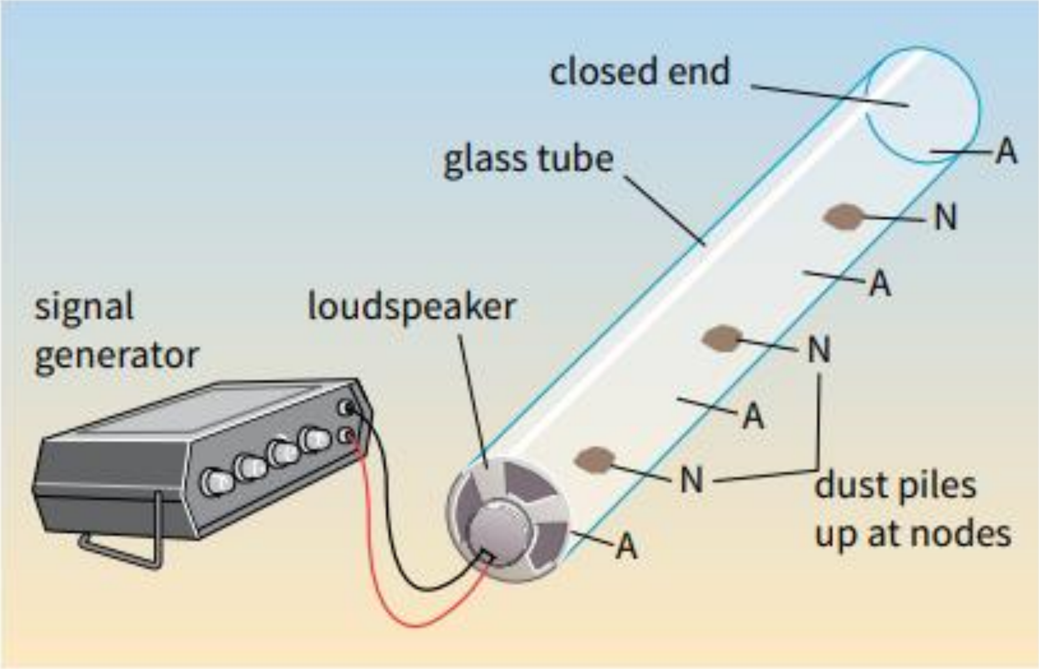


Figure 15.16 Kundt's dust tube can be used to determine the speed of sound.

Using stationary sound waves to determine wavelength and Velocity

- This is same arrangement as used for microwaves.
- The loudspeaker produces sound waves and these are deflected from the vertical board.
- The microphone detects the stationary sound wave in the space between the speaker and the board its output is displayed on the oscilloscope.
- The height of the vertical trace gives a measure of the intensity of the sound.
- By moving the microphone along the line between the speaker and the board, it is easy to detect nodes and antinodes.

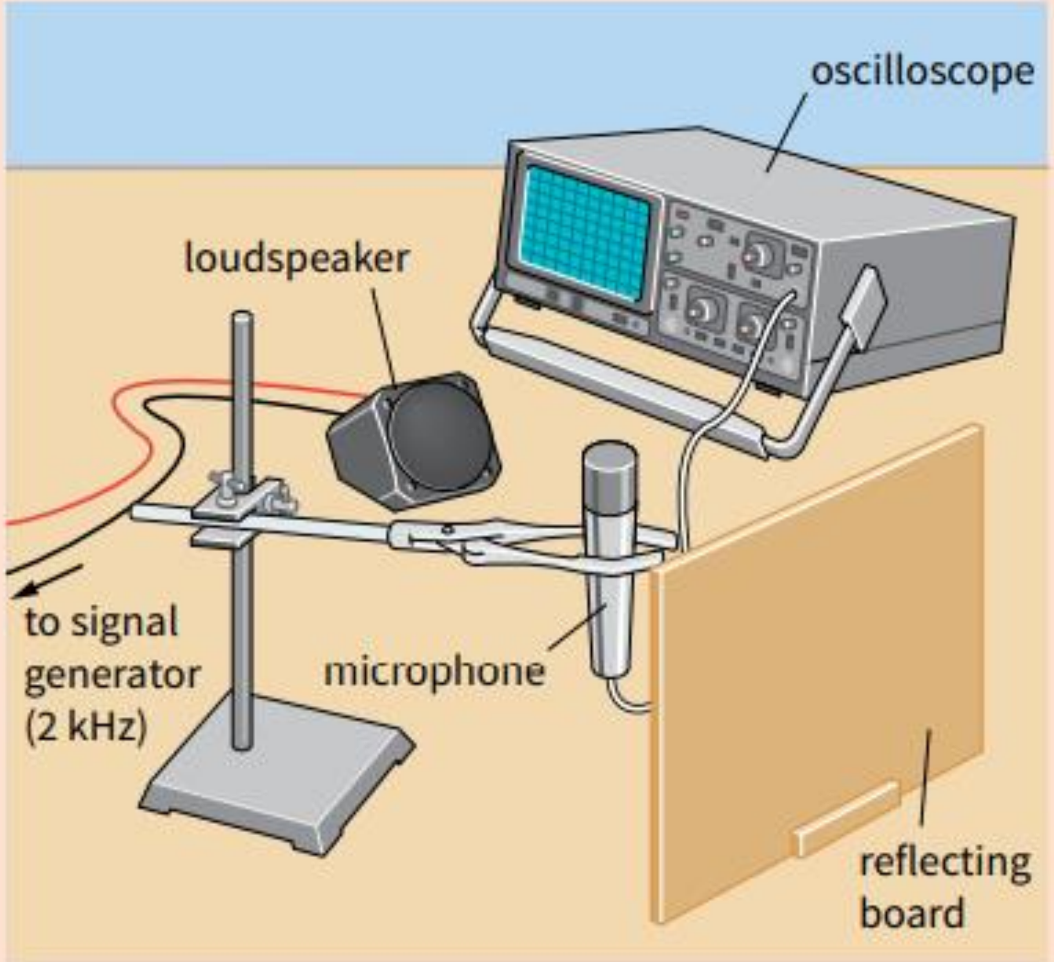


Figure 15.17 A stationary sound wave is established between the loudspeaker and the board.

Reducing and eliminating errors

- The resonance tube experiment can be used to determine the wavelength and speed of sound with a high degree of accuracy.
- In order to produce a stationary wave, there must be a minima (node) at one end and a maxima (antinode) at the end. But
- In organ pipe, we take the antinode exactly at the open end and calculation with L is made. However, the anti-nodes lies a little outside the open end due to air just outside the open end is set into vibration.
- The distance between the antinode and the open end of the pipe is called end correction.

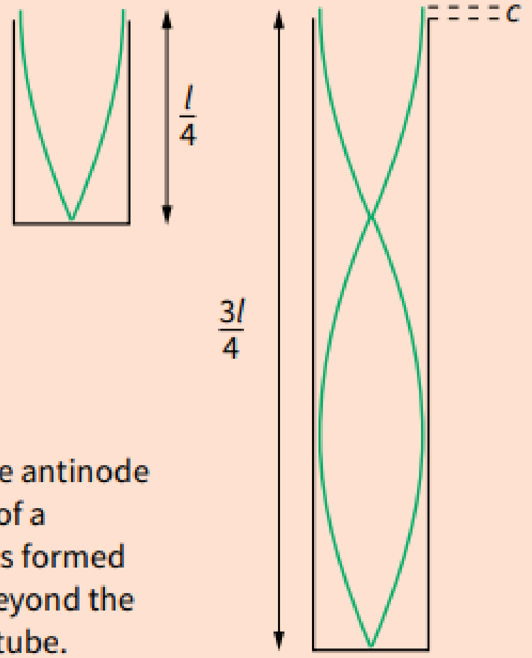


Figure 15.18 The antinode at the open end of a resonance tube is formed at a distance *c* beyond the open end of the tube.

The antinode is at a distance c beyond the end of the tube, where c is called the **end-correction**. Unfortunately, we do not know the value of c. It cannot be measured directly. However, we can write:

for the shorter tube, $\frac{\lambda}{4} = l_1 + c$

for the longer tube, $\frac{3\lambda}{4} = l_2 + c$

Subtracting the first equation from the second equation gives:

$$\frac{3\lambda}{4} - \frac{\lambda}{4} = (l_2 + c) - (l_1 + c)$$

Simplifying gives:

$$\frac{\lambda}{2} = l_2 - l_1$$

Hence:

$$\lambda = 2(l_2 - l_1)$$

- Although we do not know the value to C, we can make two measurement (l_1 and l_2) and obtain an accurate value of λ.
- The end correction C is an example of a systematic error.
- For one end open organ pipe only one end correction C.
- For both open end pipe the end correction is 2C

A student wants to demonstrate a stationary wave.

What could they use to produce this?

A passing water waves through a narrow slit

B passing monochromatic light through a double slit

C making a loud sound near a mountain

D blowing air over the top of an empty bottle

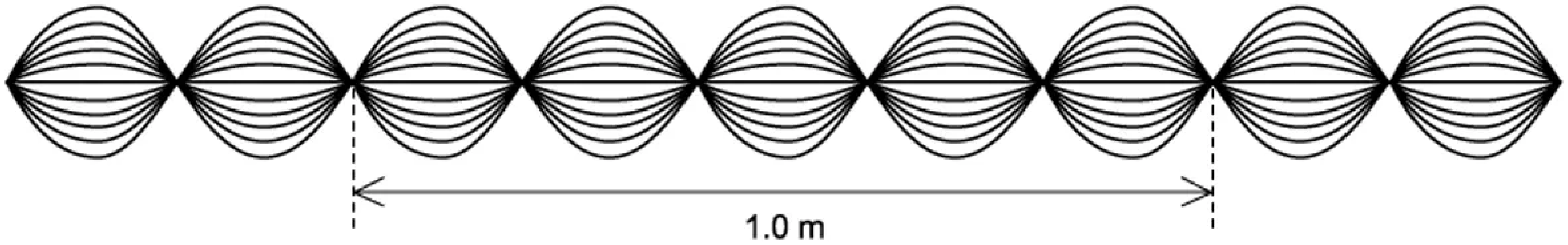
Headphones that have noise reduction can cancel out external noise by producing their own waves.

They are fitted with a microphone that detects the external sound frequency. The loudspeaker then produces a wave with the same frequency but different phase.

What would the phase difference needed to cancel out the external sound?

A 360° **B** 270° **C** 180° **D** 90°

The diagram below shows a wave pattern over time



What is the correct description of the wave?

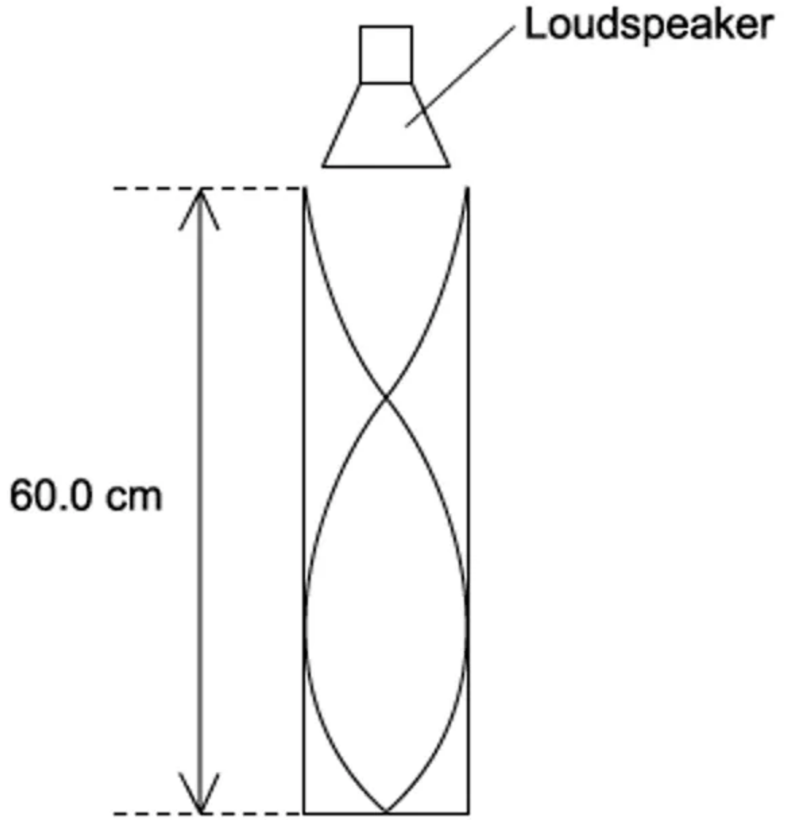
A the wave is transverse, has a wavelength of 40cm and is stationary

B the wave is transverse, has a wavelength of 40cm and is progressive

C the wave is transverse, has a wavelength of 20 cm and is stationary

D the wave is longitudinal, has a wavelength of 20 cm and is stationary

The diagram below shows sound wave produced in a column by a loudspeaker. The speed of sound in air is 330 m s^{-1}



What is the frequency of the wave?

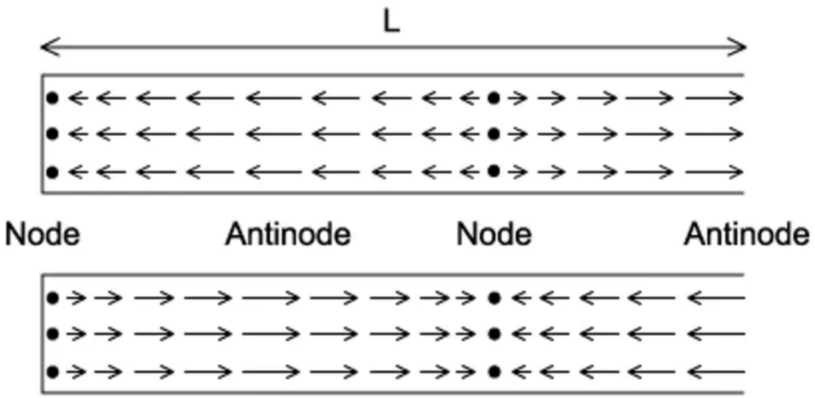
A 1650 Hz **B** 830 Hz **C** 550 Hz **D** 413 Hz

A stationary sound wave produced in a musical instrument, this instrument has the pipes open at one end and closed at the other.

If the length of the pipe was 10 m what it is the lowest frequency of sound when the speed of sound is 320 m s^{-1}?

A 32 Hz **B** 16 Hz **C** 8 Hz **D** 4 Hz

A stationary wave was formed in a column. The diagram below shows the movement of the air particles.

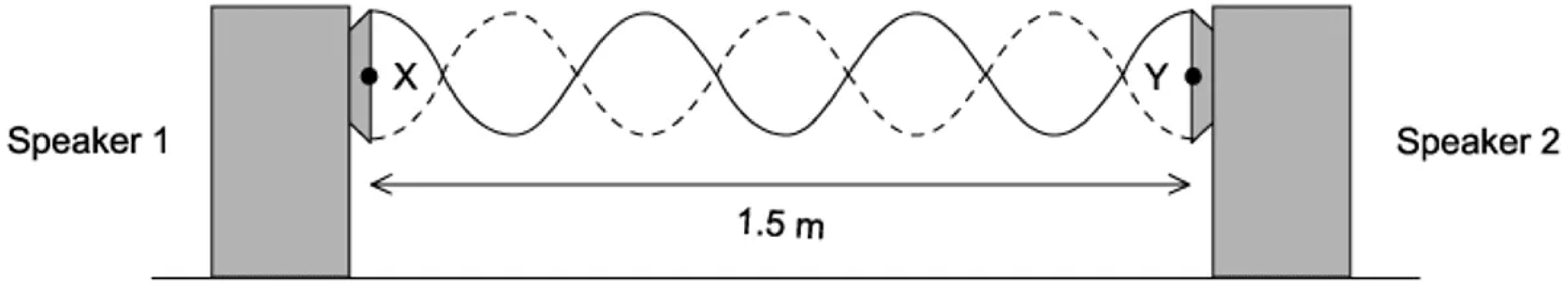


The first diagram shows the displacement and the second is the displacement half a cycle later.

Which position will the pressure change by the most significant amount, and what is the length, L of the column in terms of wavelength, λ?

	length L	maximum pressure change
A	$\frac{3}{4}\lambda$	antinode
B	$\frac{3}{4}\lambda$	node
C	$\frac{3}{2}\lambda$	antinode
D	$\frac{3}{2}\lambda$	node

Two loudspeakers emitting sound of the same frequency produces a stationary wave. This is shown in the diagram



The microphone is moved between X and Y for a distance of 1.5 m. There were six nodes and seven antinodes observed.

What is the wavelength of the sound?

A 0.21 m **B** 0.25 m **C** 0.43 m **D** 0.50 m

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Thank you!