

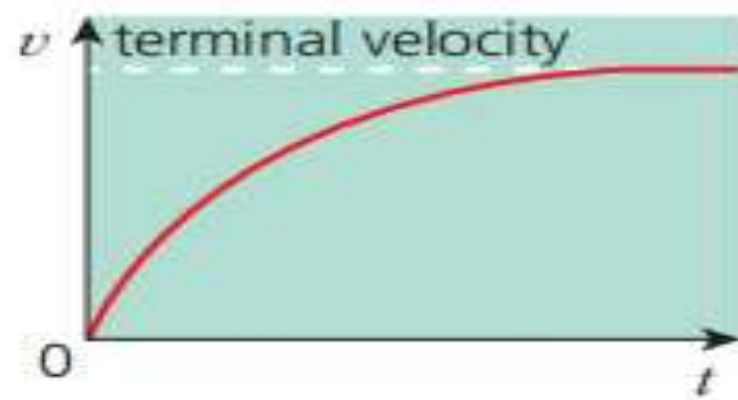
# Non- Uniform Motion

- **Friction** is the force resisting the relative motion of solid surfaces, fluid layers, and material elements sliding against each other. A frictional force always acts in the opposite direction to the relative motion of the objects.
- **Viscous force** is defined as the force between the fluid and body that are moving past in the direction for opposing the fluid flow past the object.
- We use the term, viscous force (or drag force) to describe the frictional force in a fluid (a liquid or a gas). The property of the fluid determining this force is the viscosity of the fluid.





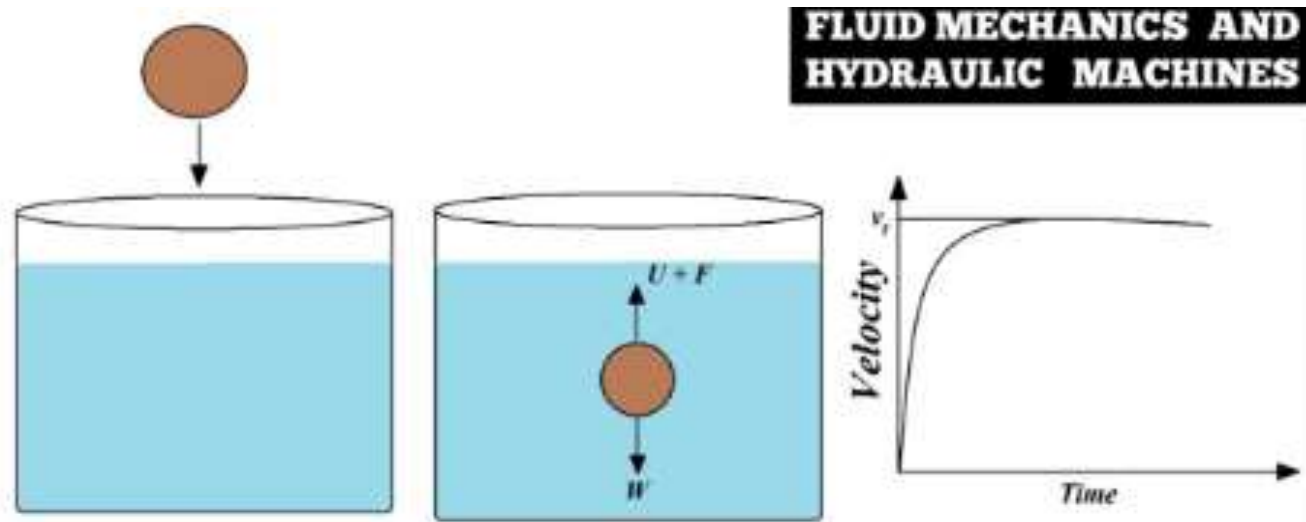
▲ **Figure 3.11** A parachutist about to land



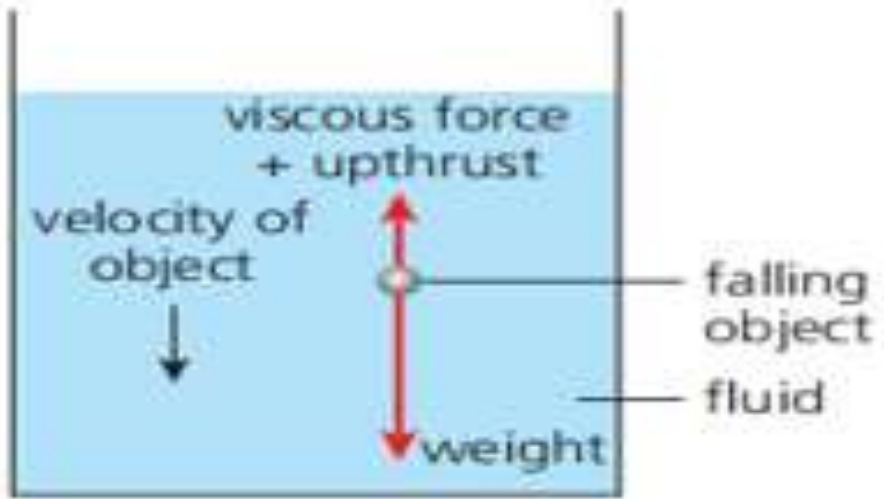
▲ **Figure 3.12** Motion of an object falling in air

- The velocity of an object moving through a resistive fluid (a liquid or a gas) does not increase continuously for ever, but eventually reaches a maximum velocity, called the terminal velocity.
- The drag force due to air resistance is zero when an object's velocity is zero but this drag force due to air resistance increases with speed.
- The acceleration is initially equal to  $g$ , but decreases to zero at the time when the terminal velocity is achieved. Thus, raindrops and parachutists are normally travelling at a constant speed by the time they approach the ground (Figure 3.11).

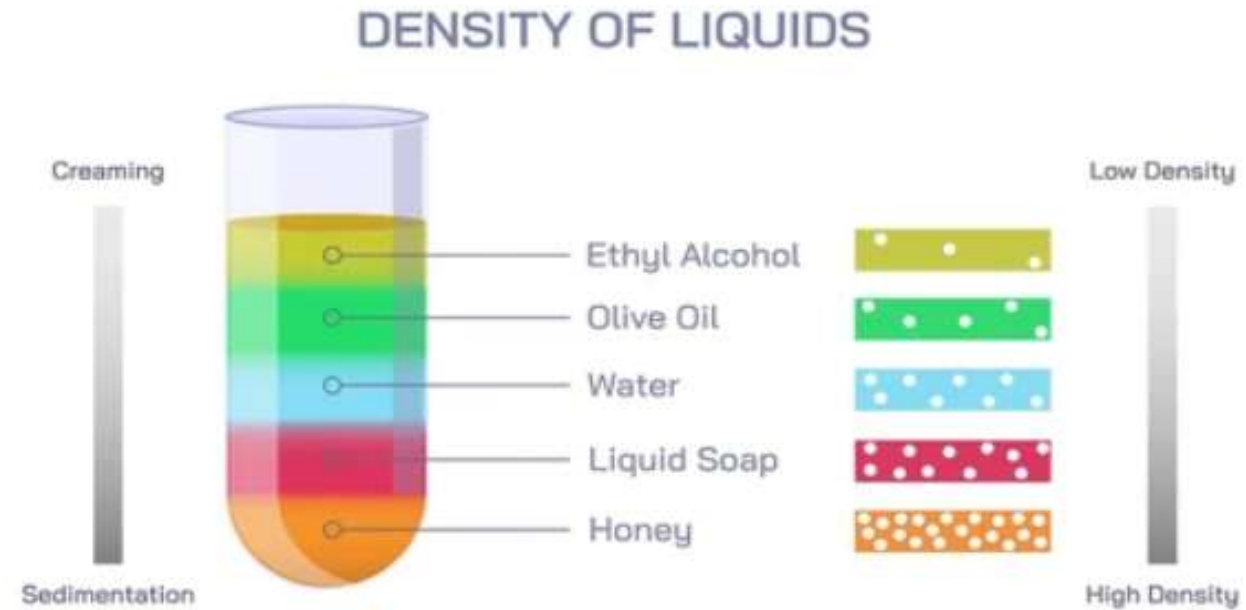
- Terminal velocity is defined as the highest velocity attained by an object falling through a fluid.



- For an object falling in a viscous fluid, the resultant downward force (weight – viscous force) decreases as the viscous force increases. When the resistive force has reached a value equal and opposite to the weight of the falling object the resultant force downwards is zero. So, the object no longer accelerates but continues at uniform velocity.
- When an object is immersed in a fluid, it experiences an upward force (upthrust or buoyancy force) due to the pressure difference of the fluid on it.



▲ **Figure 3.13 Forces on an object falling in a fluid**

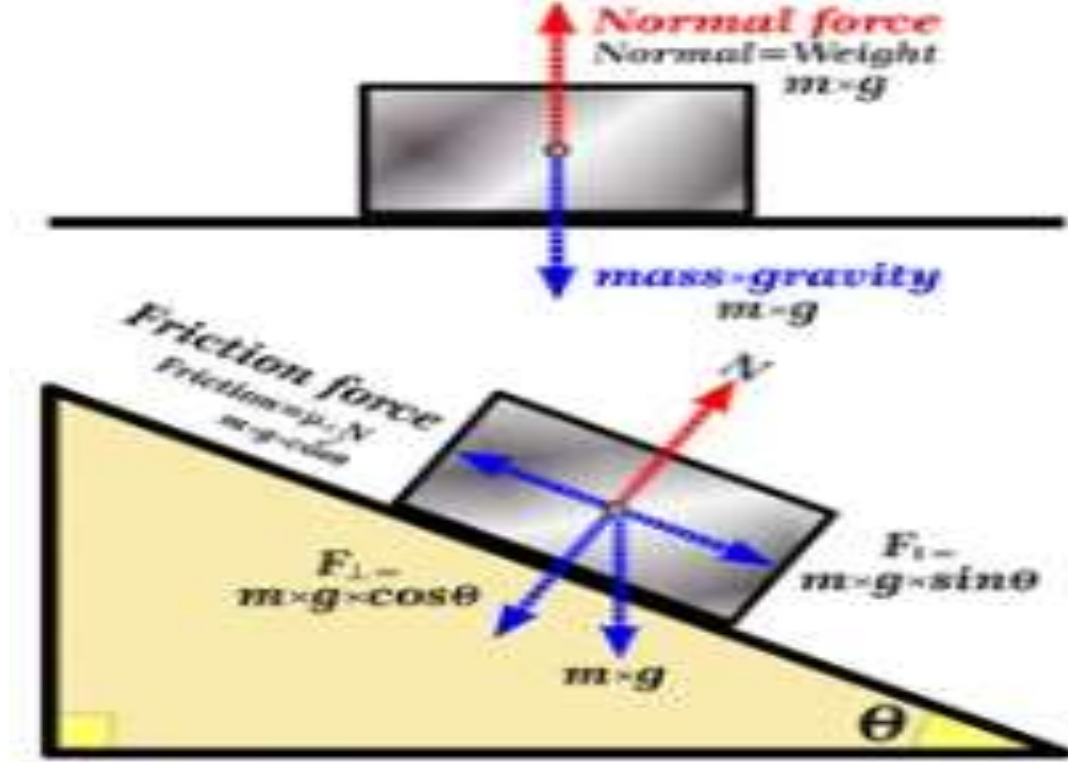
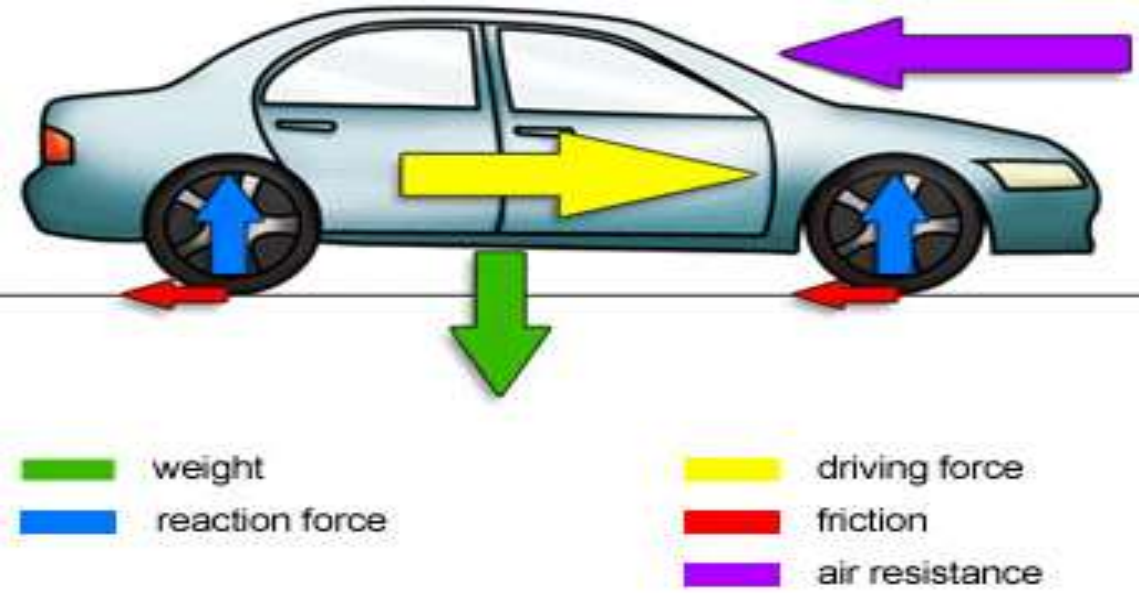


➤ The viscous force increases with the velocity of the object. The resultant downward force equals weight – (upthrust + viscous force). The object reaches terminal velocity when the upthrust and the viscous force equals its weight.

$$\text{weight} = \text{upthrust} + \text{viscous force}$$

and causes the object to accelerate until the upthrust and the viscous force equals its weight. The object then continues to fall at its terminal velocity.

# Free Body Diagram

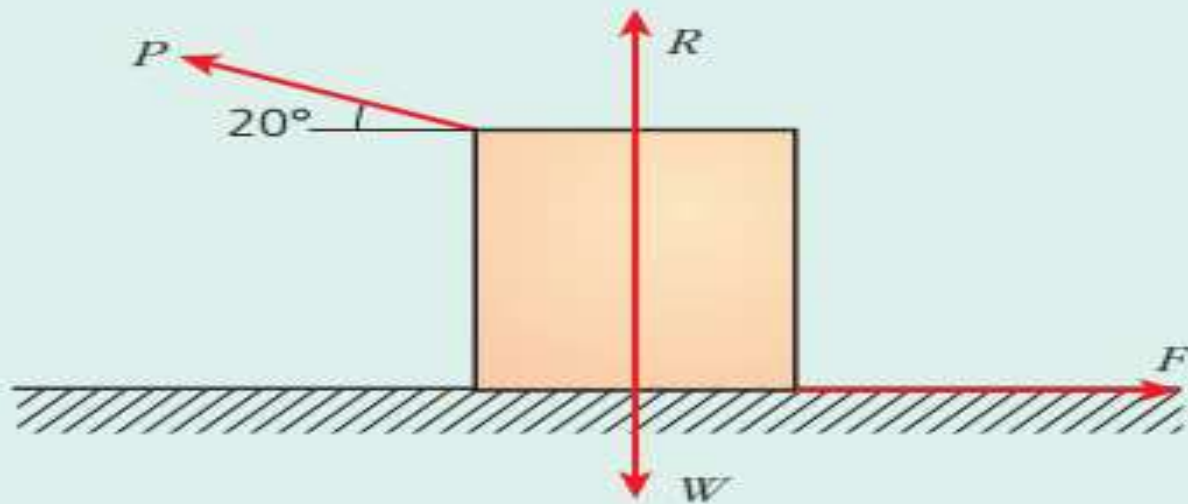


- Since many forces ( gravity, friction, resistance force due to air) are acting on the object, we can use Newton's second law to find unknown quantities.
- Newton's second law equates the resultant force acting on an object to the product of its mass and its acceleration.

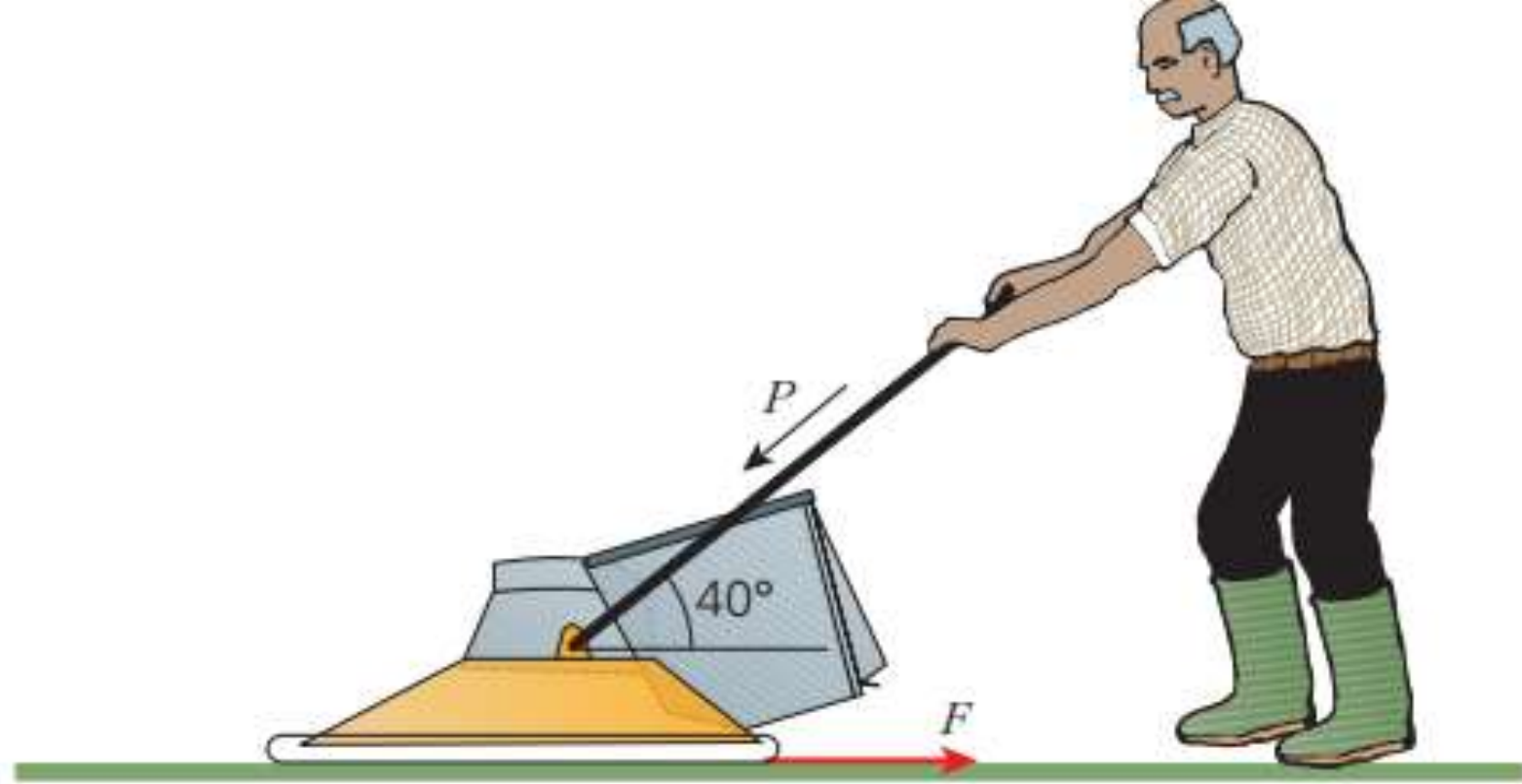
➤ In some problems, the system of objects is in equilibrium. They are at rest, or are moving in a straight line with uniform speed. In this case, the acceleration is zero, so the resultant force is also zero. In other cases, the resultant force is not zero and the objects in the system are accelerating.

- 1 A box of mass  $5.0\text{ kg}$  is pulled along a horizontal floor by a force  $P$  of  $25\text{ N}$ , applied at an angle of  $20^\circ$  to the horizontal (Figure 3.14). A frictional force  $F$  of  $20\text{ N}$  acts parallel to the floor.

Calculate the acceleration of the box.



▲ Figure 3.14



▲ **Figure 3.15**

- 3 A person gardening pushes a lawnmower of mass 18 kg at constant speed. To do this requires a force  $P$  of 80 N directed along the handle, which is at an angle of  $40^\circ$  to the horizontal (Figure 3.15).
- Calculate the horizontal frictional force  $F$  on the mower.
  - If this frictional force were constant, what force, applied along the handle, would accelerate the mower from rest to  $1.2 \text{ m s}^{-1}$  in 2.0 s?

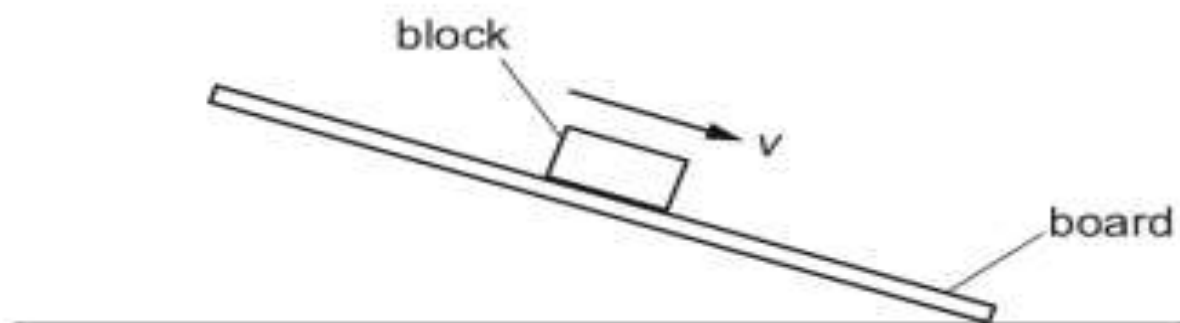
267. 9702\_m18\_qp\_12 Q: 7

A stone of mass  $m$  is dropped from a tall building. There is significant air resistance. The acceleration of free fall is  $g$ .

When the stone is falling at a constant (terminal) velocity, which information is correct?

	magnitude of the acceleration of the stone	magnitude of the force of gravity on the stone	magnitude of the force of air resistance on the stone
A	$g$	zero	$mg$
B	zero	$mg$	$mg$
C	zero	zero	$mg$
D	zero	$mg$	zero

A wooden block rests on the rough surface of a board. One end of the board is then raised until the block slides down the board at constant velocity  $v$ .



What describes the forces acting on the block when it is sliding with constant velocity?

	frictional force on block	resultant force on block
<b>A</b>	down the board	down the board
<b>B</b>	down the board	zero
<b>C</b>	up the board	down the board
<b>D</b>	up the board	zero

ridge

286. 9702\_s17\_qp\_13 Q: 10

An ice-hockey puck of mass 150g moves with an initial speed of  $2.0 \text{ m s}^{-1}$  along the surface of an ice rink.

The puck slides a distance of 30m in a straight line before stopping.

What is the average frictional force acting on the puck?

**A** 0.010 N

**B** 0.020 N

**C** 0.067 N

**D** 0.44 N

A solid sphere falls at constant (terminal) velocity in a liquid. The three forces acting on the sphere are shown in the diagram.

not to scale



How are the three forces related?

- A  $W + D = U$
- B  $W > U + D$
- C  $W - U = D$
- D  $W < D + U$

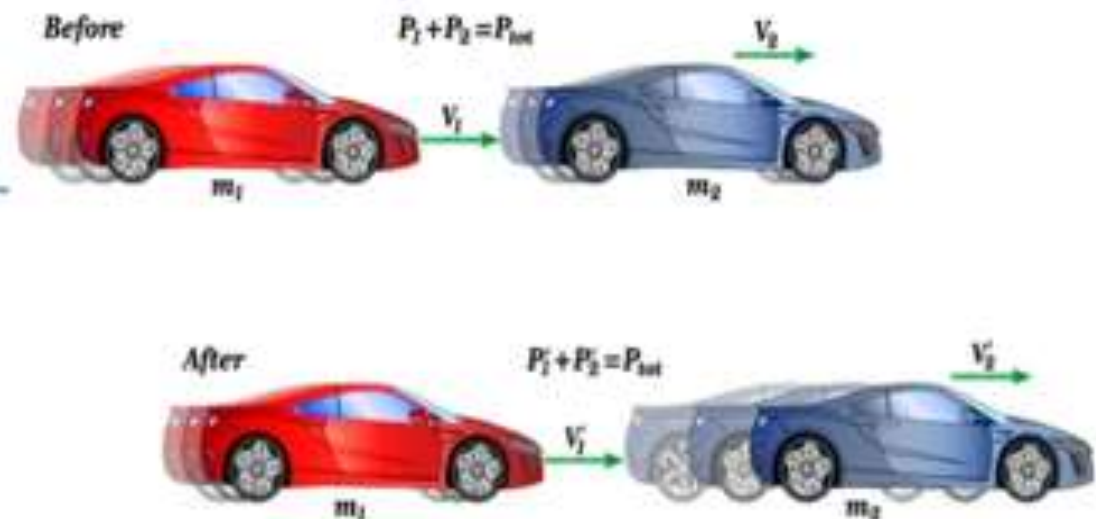
# Momentum as the product of mass and velocity

**Momentum or Linear momentum or translational momentum** is the product of the mass and velocity of an object.

**Momentum = mass x velocity**

For example, a heavy truck moving fast has a large momentum—it takes a large and prolonged force to get the truck up to this speed, and it takes a large and prolonged force to bring it to a stop afterwards. If the truck were lighter, or moving slower, then it would have less momentum.

- Like velocity, linear momentum is a vector quantity, possessing a direction as well as a magnitude:
- **Units:**  $\text{kgms}^{-1}$  or Ns



## Force as a rate of change of Momentum

- Consider a body of mass  $m$ , initially moving with a velocity of magnitude  $u$ . A force  $F$  acts on the body and causes it to accelerate to a final velocity of magnitude  $v$ .

- We can write Newton's second law in the form

$$F = m \left( \frac{v - u}{t} \right)$$

- and a simple rearrangement shows the relation between force and momentum

$$F = \frac{mv - mu}{t}$$

Remember, momentum = mass x velocity.

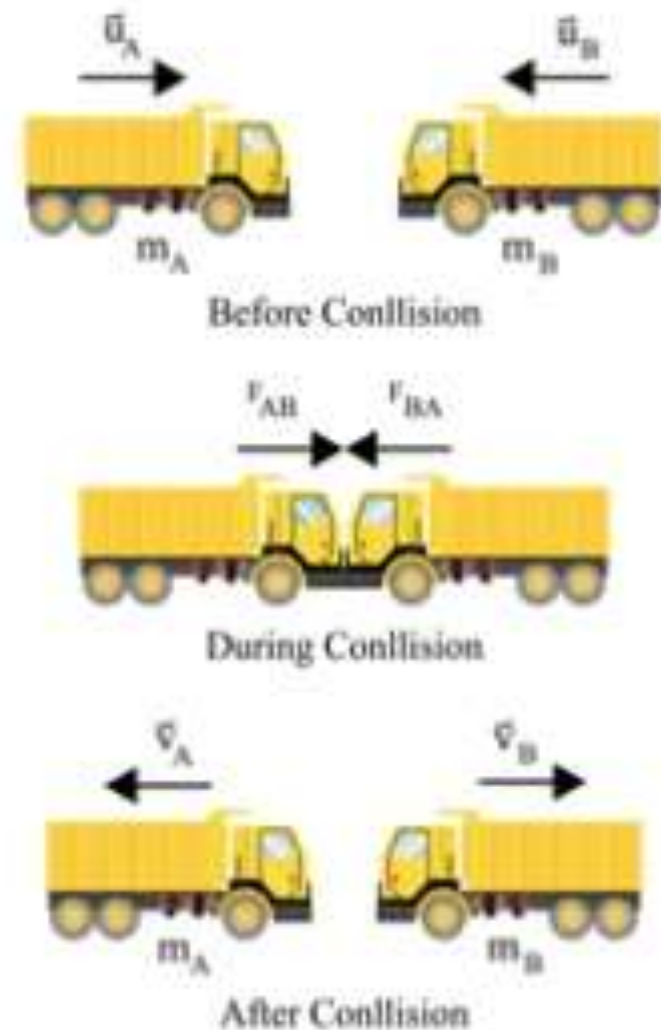
- Now,  $mv$  is the **final momentum** of the body and  $mu$  is the **initial momentum** of the body. Therefore, we have

Force = rate of change of momentum

$$F = \frac{\Delta p}{\Delta t}$$

## Principle of Conservation of Momentum

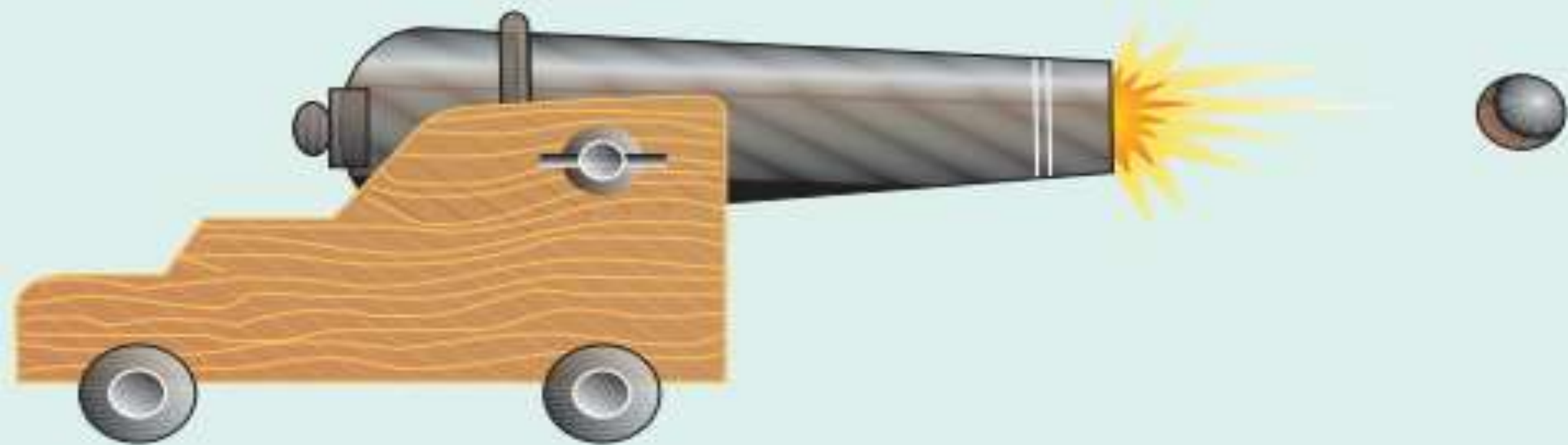
- **The Principle of the Conservation of Momentum states that:** if objects collide, the total momentum before the collision is the same as the total momentum after the collision (provided that no external forces - for example, friction - act on the system).
- Of course, energy is also conserved in any collision, but it isn't always conserved in the form of kinetic energy.



➤ An isolated system is one on which no external resultant force acts.

A cannon of mass  $1.5 \text{ tonnes}$  ( $1.5 \times 10^3 \text{ kg}$ ) fires a cannon-ball of mass  $5.0 \text{ kg}$  (Figure 3.18).

The speed with which the ball leaves the cannon is  $70 \text{ m s}^{-1}$  relative to the Earth. What is the initial speed of recoil of the cannon?



## Case 1

- To do any calculations for momentum, there are some simple rules to follow to make it easy:
- Always decide which direction is positive and which is negative, then stick to it.
- Always remember that the total momentum before the collision will be the same as the total momentum after the collision.
- So,

If these two objects collide



Then the result could be



- **The conservation of momentum states:**
- $\text{Momentum}_{\text{before}} = \text{Momentum}_{\text{after}}$
- So,  $(P_1 + P_2)_{\text{before}} = (P_1 + P_2)_{\text{after}}$
- Or,  $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$
- But notice that in this example,  $v_1 = 0$ . So that term cancels and makes finding an answer much easier.

## Case 2

- If the objects change direction in the collision or are going in different directions before the collision, make sure that you have got the signs for the velocities and therefore the momentums correct.

### Example 1

If these two objects collide



Because the objects are moving in opposite directions, we have to treat one of the velocities as negative. And so:

$$\text{Initial } P = m_1 u_1 + (-m_2 u_2)$$

### Example 2

Initially:



Becomes, after a collision:

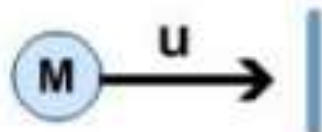


Note that the direction and sign of velocity (and therefore momentum) of M1 changes after the collision.

### Case 3

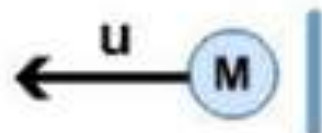
- When objects **bounce back** after a collision, be careful about the change in momentum.

*Example* Initially:



Then there is an elastic collision (ie  $v$  and  $u$  have the same magnitude but opposite directions)

Finally:



So change in momentum = final P - initial P

$$= -mu - (+ mu)$$

$$= -2mu$$

# Explosions

- Explosions are a special type of collision. Momentum is conserved in an explosion. This is made easier by the fact that usually, the momentum **before** an explosion is **zero**. The Principle of the Conservation of Momentum states that the momentum **after** the explosion must therefore be **zero** as well.
- **What's the momentum of the universe?**

## So what is its momentum ?

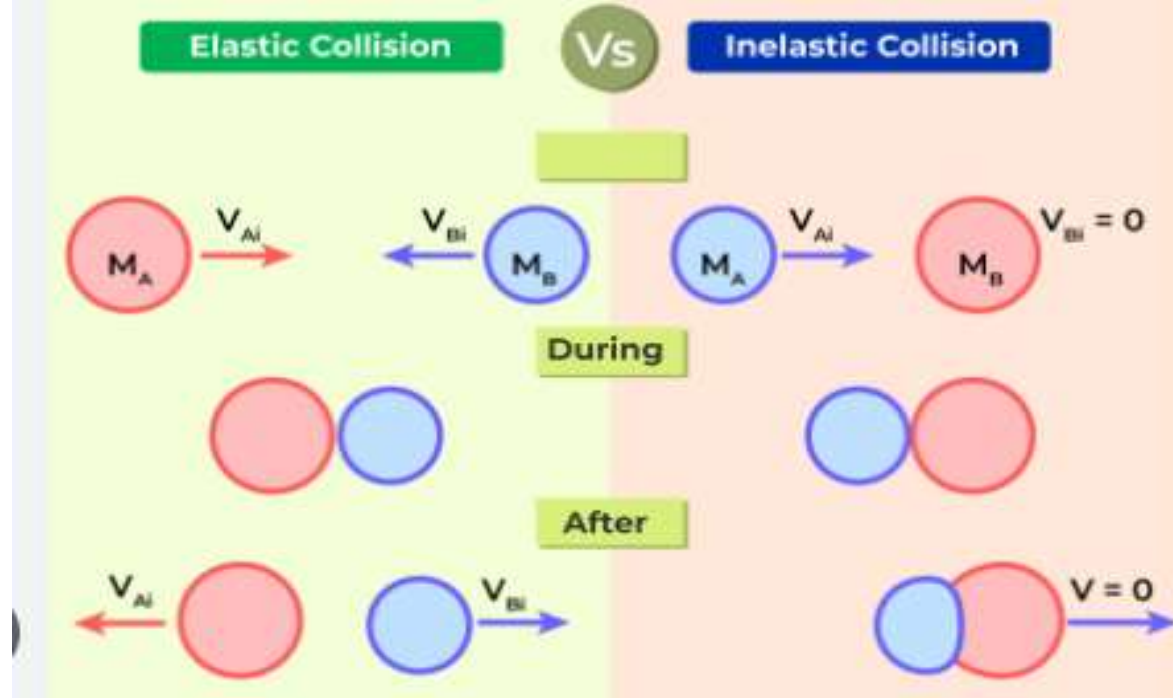
Force can be defined as the rate of change of momentum as:

$$F = ma$$

$$\text{But, } a = \frac{v - u}{t}$$

$$\text{So, } F = m \left( \frac{v - u}{t} \right) = \frac{mv - mu}{t} = \frac{\text{change in momentum}}{\text{time}}$$





- An elastic collision is one in which the total kinetic energy remains constant. In this situation, the relative speed of approach is equal to the relative speed of separation.
- An inelastic collision is one in which the total kinetic energy is not the same before and after the event.
- Although kinetic energy may or may not be conserved in a collision, momentum is always conserved, and so is total energy.

## Principle of Conservation of Momentum

### Elastic and Inelastic collisions

- **Perfectly Elastic collisions**
- All momentum is conserved
- Kinetic energy is conserved as well.
- Relative speed of approach = relative speed of separation.
- (So if one is catching the other at  $10\text{m/s}$  before the collision, it will be moving apart from it at  $10\text{m/s}$  after the collision)
- **Perfectly Elastic** collisions are surprisingly common. All collisions between atoms are Perfectly Elastic according to the Kinetic Theory of Gases.

## ELASTIC COLLISION



# Principle of Conservation of Momentum

## Elastic and Inelastic collisions

### Perfectly Inelastic collisions

All momentum is conserved (as always).

Kinetic energy is **not** conserved.

The relative speed of separation is zero.

(In other words, that means the objects stick together after the collision, they will move together, so just consider them as one object whose mass is the same as that of the two original masses combined).

### INELASTIC COLLISION



perfectly inelastic collisions  
total momentum is conserved



kinetic energy  
is not conserved

## Why is kinetic energy not conserved while momentum is conserved in a perfectly inelastic collision?

- It goes into heat, sound, work done to deform the colliding bodies etc. Other forms of energy, in other words.
- Momentum is not a type of energy. Momentum and energy are totally different physical quantities with different physical dimensions. (Energy is the capacity to do work)

Conservation of momentum in a system occurs provided that there are no *external* forces acting on a system. This is a consequence of Newton's 2nd law and Newton's 3rd law.

Newton's 2nd law says that the net force acting on a body is equal to the rate of change of its momentum.

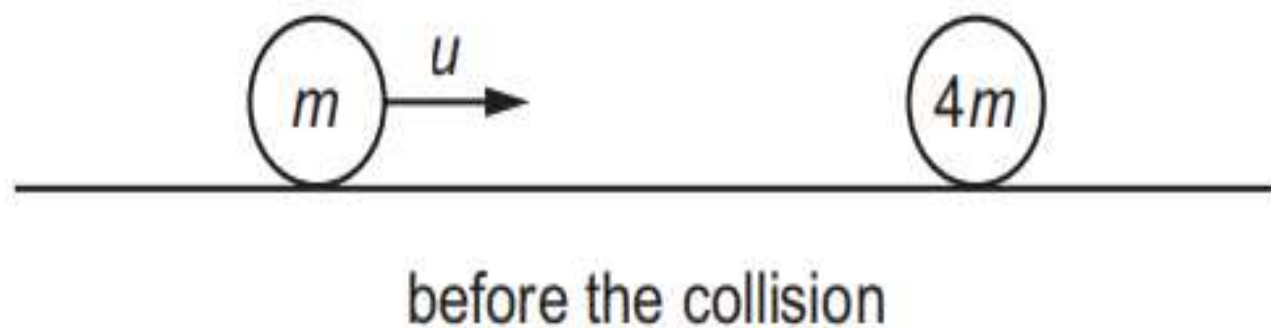
This is the full, general statement of the 2nd law.  $F = \Delta p / \Delta t$ .

If the mass of the body is constant, this reduces to  $F = m(\Delta v / \Delta t) = ma$ . Therefore, if a net force acts on an object, its momentum will change with time. If there is no net force, then its momentum will not change. |

## How and when momentum is conserved ?

- Now, consider a system of interacting particles. The particles are moving around randomly. Every once in a while, two particles (1 and 2) may collide. While this is happening, particle 1 exerts a force on particle 2. However, Newton's 3rd law says that particle 2 must therefore, at the same time, exert a force on particle 1 of equal strength and opposite direction. These forces are also exerted over the same time interval (while the particles are in contact). Therefore, the change in momentum of particle 1 will be equal in magnitude and opposite in direction to the change in momentum of particle 2.
- These two momentum changes therefore cancel each other out. Each particle may individually change its momentum, but there will be no change to the *total* momentum of the system. In other words, since Newton's 3rd says that these *internal* forces always occur in matched "action-reaction" pairs, they cannot ever cause a change to the overall momentum of the system. Only an *external* force (a force from something that is not part of the system of particles) can cause a change in the total momentum of the system. In the absence of a net external force,  $F_{\text{tot}} = 0$  and hence  $\Delta p_{\text{tot}} = 0$ . In the absence of external forces, momentum is *conserved*.

An object of mass  $m$ , moving at speed  $u$  along a frictionless horizontal surface, collides head-on with a stationary object of mass  $4m$ .



After the collision, the object of mass  $m$  rebounds along its initial path with  $\frac{1}{4}$  of its kinetic energy before the collision.

What is the speed of the object of mass  $4m$  after the collision?

- A**  $\frac{u}{8}$       **B**  $\frac{3u}{16}$       **C**  $\frac{5u}{16}$       **D**  $\frac{3u}{8}$

Skaters of masses 80 kg and 40 kg move directly towards each other and collide.

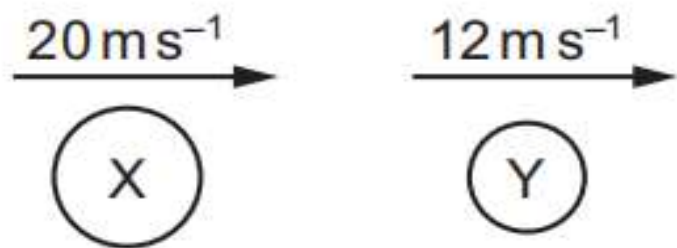
Before the collision, the heavier skater is moving to the right at a speed of  $2.0 \text{ m s}^{-1}$  and the lighter skater is moving to the left at a speed of  $1.0 \text{ m s}^{-1}$ .

After the collision, the heavier skater moves to the right at a speed of  $0.80 \text{ m s}^{-1}$ .

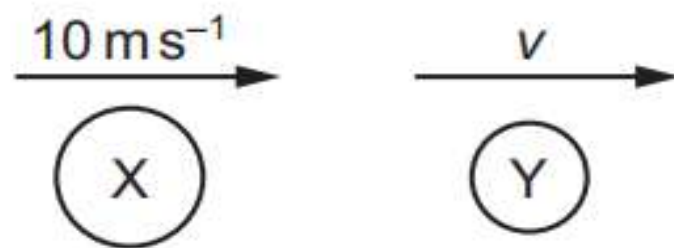
What is the relative speed of separation of the two skaters?

- A**  $0.6 \text{ m s}^{-1}$       **B**  $1.4 \text{ m s}^{-1}$       **C**  $2.2 \text{ m s}^{-1}$       **D**  $2.6 \text{ m s}^{-1}$

Two objects X and Y in an isolated system undergo a perfectly elastic collision. The velocities of the objects before and after the collision are shown.



before  
collision

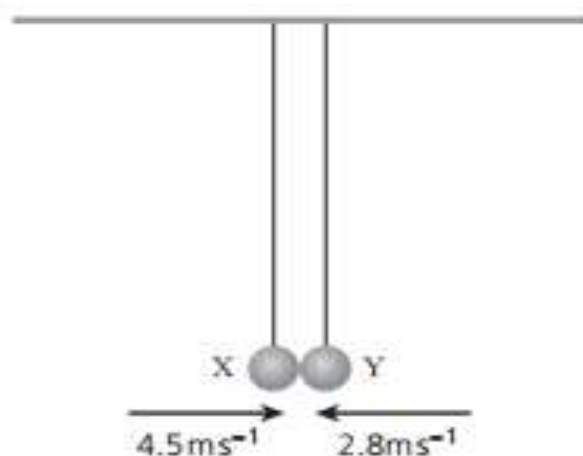


after  
collision

What is the speed  $v$  of Y after the collision?

- A**  $2.0 \text{ m s}^{-1}$       **B**  $18 \text{ m s}^{-1}$       **C**  $22 \text{ m s}^{-1}$       **D**  $24 \text{ m s}^{-1}$

21 Two balls X and Y are supported by long strings, as shown in Fig. 3.29.



▲ Figure 3.29

The balls are each pulled back and pushed towards each other. When the balls collide at the position shown in Fig. 3.29, the strings are vertical. The balls rebound in opposite directions. Fig. 3.30 shows data for X and Y during this collision.

ball	mass	velocity just before collision/ $\text{m s}^{-1}$	velocity just after collision/ $\text{m s}^{-1}$
X	50 g	+4.5	-1.8
Y	M	-2.8	+1.4

▲ Figure 3.30

The positive direction is horizontal and to the right.

- Use the conservation of linear momentum to determine the mass  $M$  of Y. [3]
- State and explain whether the collision is elastic. [1]
- Use Newton's second and third laws to explain why the magnitude of the change in momentum of each ball is the same. [3]

- 4 (a) State the principle of conservation of momentum.

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..... [2]

- (b) Two balls, X and Y, move along a horizontal frictionless surface, as shown from above in Fig. 4.1.

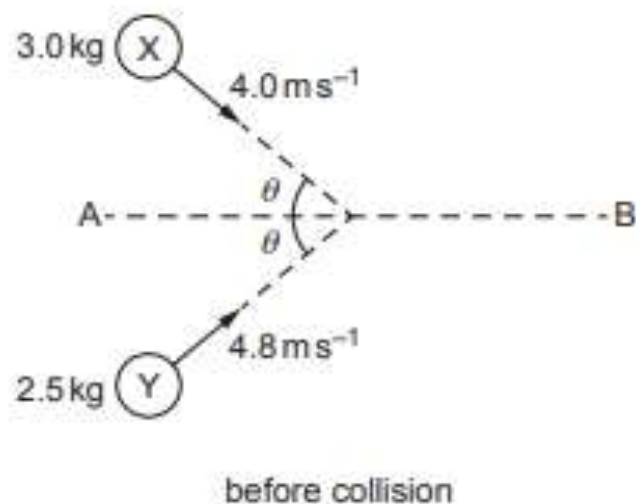


Fig. 4.1 (not to scale)

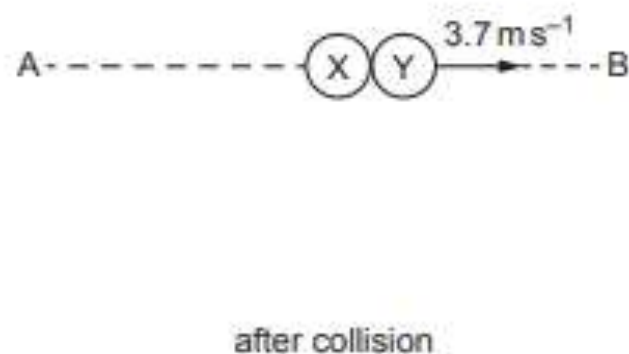


Fig. 4.2 (not to scale)

Ball X has a mass of 3.0 kg and a velocity of  $4.0 \text{ ms}^{-1}$  in a direction at angle  $\theta$  to a line AB. Ball Y has a mass of 2.5 kg and a velocity of  $4.8 \text{ ms}^{-1}$  in a direction at angle  $\theta$  to the line AB.

The balls collide and stick together. After colliding, the balls have a velocity of  $3.7 \text{ ms}^{-1}$  along the line AB on the horizontal surface, as shown in Fig. 4.2.

- (i) By considering the components of the momenta along the line AB, calculate  $\theta$ .

# Impulse

- The impulse of a force  $F$  is the product of the force and the time  $\Delta t$  for which it acts:

$$\text{Impulse} = F\Delta t$$

- The impulse of a force acting on an object is equal to the change of momentum of the object:  $F\Delta t = \Delta p$ .
- The unit of impulse is  $\text{Ns}$ .

- 7 A golfer hits a ball of mass  $45\text{ g}$  at a speed of  $40\text{ m s}^{-1}$  (Figure 3.20). The golf club is in contact with the ball for  $3.0\text{ ms}$ . Calculate the average force exerted by the club on the ball.



▲ Figure 3.20