Graph

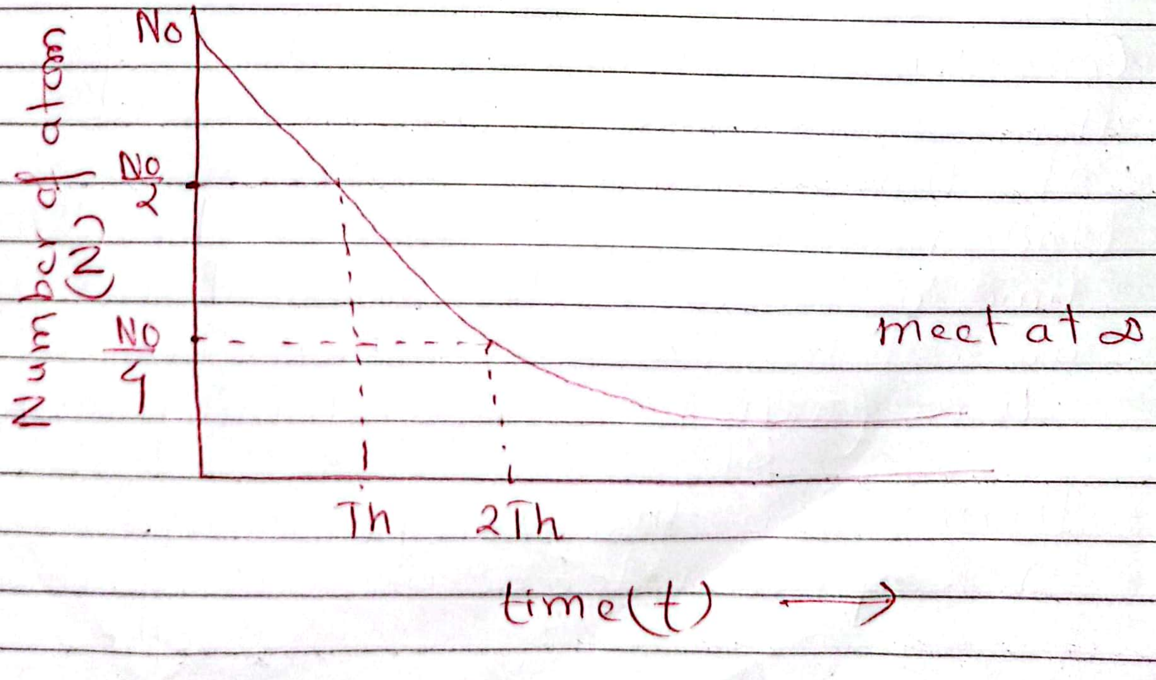
Question No. 1

(a) Name the graph shown in figure

(b) Derive $N = N_0 e^{-\lambda t}$ in radioactive decay law

(c) A sample of radioactive isotopes contain 50 % of its original number in 2 years
Then
(i) What is its half life?
(ii) If there are 10^6 such nuclei remaining after 8 years, how many numbers are there in the begening

(d) What will be the value of number of atom when time becom 3Th?



(a) Ans

The graph shown in the figure is the graph of number of atom (N) against time(t) where no of atom decreases exponentiall with time
It means (rapidly at first and slowly) afterward.

(b) Ans

The law of radioactive decay states that "The rate of disintegration of radioactive substance at any instant is directly proportional to number of atom left (undecayed) at that instant.

Consider No be the initial no of atoms of a radioactive substance at time (t=0) and N be the number of atoms left after time 't'. If 'dN' atom disintegrates in time 'dt' then

Here $\left(-\frac{dN}{dT}\right)$ is rate of integration

Then, from the law of disintegration
rate of disintegration ∝ No of atom left

$$\left(-\frac{dN}{dt}\right) \propto N$$

$$\frac{dN}{dt} = -\lambda N \quad -(1)$$

λ is disintegration or decay constant and -ve sign show number of parent atoms are decreasing with time

then eqn (1) can be written as

$$\frac{dN}{N} = -\lambda dt$$

$$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_{0}^{t} dt$$

$$[\ln N]_{N_0}^{N} = -\lambda [t]_0^t$$

$$\ln N - \ln N_0 = -\lambda t$$

$$\ln \frac{N}{N_0} = -\lambda t$$

taking anti log

$$\frac{N}{N_0} = e^{-\lambda t}$$

$$\boxed{N = N_0 e^{-\lambda t}}$$

(C) Given time $(t) = 2$ year

let N_0 be the initial number of atom

Then N be the no of atom left in time 't'

~~Atq~~ According to question

50% of the isotopes remains of initial number then

(a) $N = 50\%$ of N_0

$N = \frac{50}{100} \times N_0$

$N = \left(\frac{1}{2}\right) N_0$

$\therefore \frac{N}{N_0} = \frac{1}{2}$

It (i) its half life $T_{\frac{1}{2}} =$

(b) $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T_{\frac{1}{2}}}$

$\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^{2/T_{\frac{1}{2}}}$

$1 = \frac{2}{T_{\frac{1}{2}}}$

$\therefore T_{\frac{1}{2}} = 2$ years

(ii) No of nuclei remaining $(N) = 10^6$

time $(t) = 8$ years

half life $= 2$ years

we have

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T_{\frac{1}{2}}}$$

$\frac{10^6}{N_0} = \left(\frac{1}{2}\right)^{8/2}$

$\frac{10^6}{N_0} = \frac{1}{16}$

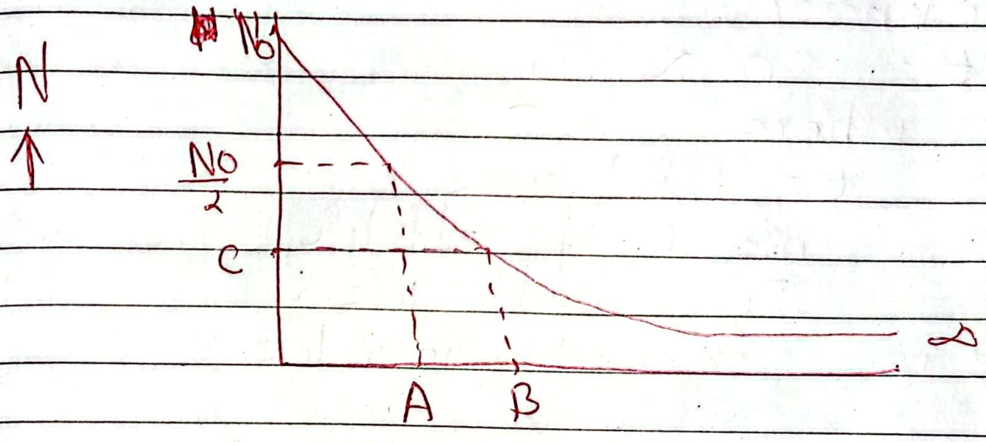
$\therefore N_0 = 1.6 \times 10^7$ atoms

CamScanner

(d) Ans from graph we can say the no of atom when time become 3 Th is $\frac{N_0}{8}$.

Question '2'

(a) Write down the algebraic equation that represent the given curve?



(b) What is A?
(c) What is the value of C if B is second half life
(d) Derive $N = N_0 e^{-\lambda t}$ where symbols have usual meaning
(e) Define mean life and decay constant

Solution

(a) The algebraic eq^n that represents the graph is
$N = N_0 e^{-\lambda t}$
This is called decay eq^n

(b) Ans A = half life ($T_{\frac{1}{2}}$)

It is the time at which half of the radioactive substance remains undecayed from graph, Number of atoms falls to $\frac{N_0}{2}$ from N_0 which means that half of substance remains undecayed

(c) We know,

$t = n \times T_{\frac{1}{2}}$

where

t = total time

n = No of half life

$T_{\frac{1}{2}}$ = half life

As B is second half life

$n = 2$

we have

$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$

$\frac{N}{N_0} = \left(\frac{1}{2}\right)^2$

$\frac{N}{N_0} = \frac{1}{4}$

$\therefore N = \frac{N_0}{4}$

Alternative

$t = n \times T_{\frac{1}{2}}$

$= 2 T_{\frac{1}{2}}$

$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T_{1/2}}$

$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{2T_{\frac{1}{2}}/T_{\frac{1}{2}}}$

$\frac{N}{N_0} = \frac{1}{4}$

$\therefore N = \frac{N_0}{4}$

$\therefore \boxed{C = \frac{N_0}{4}}$

(d) Do your self

(e) Mean life

The mean or average life of radioactive substance is defined as the ratio of sum of life of all radioactive atom to the total no of atom present initially.

Mathematically

$$\bar{T} = \frac{1}{\lambda}$$

It is the reciprocal of decay constant

Decay constant

we have $\frac{dN}{dt} = \lambda N$ [In magnitude]

$$\lambda = \frac{\left(\frac{dN}{dt}\right)}{N}$$

It is defined as the ratio of disintegration at instant to number of atom left at that ~~constant~~ instant.

Also from decay eq^n

$$N = N_0 e^{-\lambda t}$$

If $\lambda = \frac{1}{t}$ then $N_0 = N_0 e^{-1}$

$$N = \frac{1}{e} N_0$$

or, $N = \frac{N_0}{2.718}$ $\therefore N = 37\% \cdot N_0$

Hence decay constant can be defined as the reciprocal of time during which the number of atoms of a radioactive substance falls to 37% of its initial value. (ie decreases by 63%.)

Derive Relation between $T_{\frac{1}{2}}$ & $\bar{T}$

$$T_{\frac{1}{2}} = \frac{0.6931}{\lambda} \quad (1)$$

$$\bar{T} = \frac{1}{\lambda}$$

$$\therefore T_{\frac{1}{2}} = 0.6931 \times \bar{T}$$

$$\bar{T} = \frac{1}{0.6931} \times T_{\frac{1}{2}}$$

$$\bar{T} = 1.443 \times T_{\frac{1}{2}}$$