XAVIER
INTERNATIONAL COLLEGE

GRADE XII

Physics
Xavier International College

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Chapter-3 Fluid Statics

XAVIER INTERNATIONAL COLLEGE

Teaching period= 9

Key Points

Fluid Statics

A substance that can flow from one point to another is called the fluid. Like Liquid and gas.

Fluid Statics: The branch of fluid mechanics that deal with the behavior of fluid when they are at rest.

This includes two situations

i) When the fluid is at rest

ii) when it moves like a rigid solid

Density and Relative Density: The density of a substance is defined as the mass per unit volume. The relative density of a substance is the ratio of its density to the density of water at $4^0\ c$.

$$\text{Relative density, } \rho_r = \frac{\rho}{\rho_w}$$

Pressure

The perpendicular force exerted by a fluid per unit area.

This is equivalent to stress in solids, but we shall keep the term pressure. Mathematically, because pressur may vary from place to place, we have:

$$p = \lim_{\Delta \to 0} \frac{\Delta F}{\Delta A}$$

As we saw, force per unit area is measured in N/m2 which is the same as a pascal

(Pa). The units used in practice vary:

1 kPa = 1000 Pa = 1000 N/m2

1 MPa = 1000 kPa = 1 ×106 N/m2

1 bar = 105 Pa= 100 kPa = 0.1 MPa

1 atm = 101,325 Pa = 101.325 kPa = 1.01325 bars = 1013.25 millibars.

Pascal's Law of Pressure

- This law states: When a pressure is applied to an enclosed liquid, the pressure is equally transmitted to every portion of it.
- This properties is used in a hydraulic system to produce larger force from a smaller one.

Application of Pascal's Law

The French scientist Blaise Pascal observed that the pressure in a fluid at rest is the same at all points if they are at the same height. Statement of Pascal's law is If the pressure in a liquid is changed at a particular point, the change is transmitted to the entire liquid without being diminished in magnitude

Type equation here.Two cylinders A and B connected to each other by a horizontal pipe, filled with a liquid (Figure). They are fitted with frictionless pistons of cross sectional areas A_1 and A_2 ($A_2 > A_1$). Suppose a downward force F is applied on the smaller piston, the pressure of the liquid under this piston increases to P(where, $p=F_1/A_1$).But according to Pascal's law, this increased pressure P is transmitted undiminished in all directions. So a pressure is exerted on piston B. Upward force on piston B is

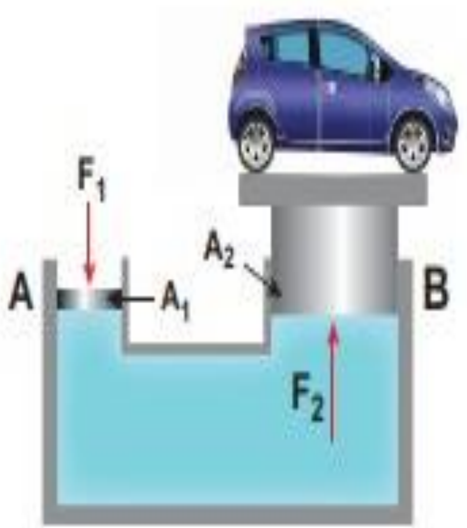


Figure 7.12 Hydraulic lift

$$p = \frac{F_1}{A_1} = \frac{F_2}{A_2} \qquad\qquad F_2 = \frac{A_2}{A_1} F_1$$

ii) Hydraulic jack: Cars and heavy trucks are raised to convenient heights by the use of hydraulic jack.

iii) Hydraulic Brakes: When a small force is applied by the foot on the brake plate, the applied Pressure is transmitted through the brake oil to act on larger area where pistons are made to move the brake shoes against the brake drums.

iv) High pressure water jet cutting: stones, slates, rubbers, forms etc. are cut by high pressure of water jet such as a pressure of 350 atmospheres and this is dust free technique.

v) Teeth scaling : On teeth scaling, tooth are hit by fine jet of water at high pressure.

Archimedes' Principle

Statement: When a body is fully or partially immersed in a fluid, it experiences an upthrust which is equal to the weight of the fluid displaced by the body.

Suppose a body has weight of W newton in air and volume V. If the body is immersed in a liquid. let the weight of the body in it be W_1. Then

Loss in weight of the body in liquid = $W - W_1$

According to Archimedes' principle, Upthrust = weight of displaced fluid

$= W - W_1$

For an object submerged in a fluid, there is a net force on the object, because the pressure at the top and bottom of it are different

Suppose a cylindrical solid block completely immersed in a liquid of density ρ. Its top surface is at a depth h_1 and bottom surface is at h_2 and resultant force on horizontal two surfaces is cancelled.

The downward force on top, $F_1 = P_1 \times A = h_1 \rho g A$

and The upward force at bottom, $F_2 = P_2 \times A = h_2 \rho g A$

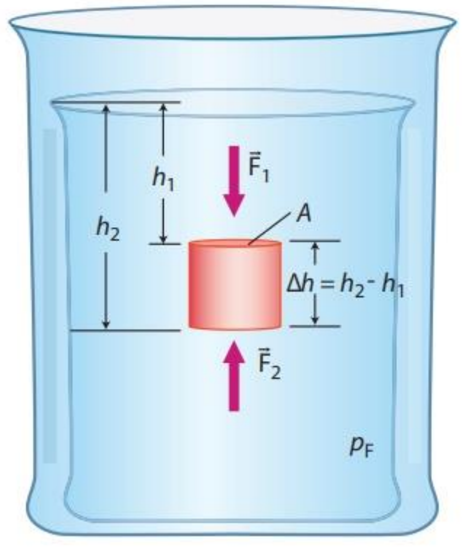


Figure 1.19 Net force acting on an object

∴ Resultant force on solid = Upward force - downward force top,

$$= F_2 - F_1$$

$$= h_2 \rho g A - h_1 \rho g A$$

$$= \rho g A(h_2 - h_1)$$

But $A(h_2 - h_1) = V$ and

Upthrust $= \rho g V$ V is the volume of liquid displaced

As **ρ V** represents the mass of displaced fluid so As **(ρ V) g** is weight of the displaced liquid.

Upthrust = weight of displaced liquid

Upthrust of Buoyancy

When a body is partially or fully immersed in a fluid, it displaces a certain amount of fluid. The displaced fluid exerts an upward force on the body. The upward force exerted by a fluid that opposes the weight of an immersed object in a fluid is called upthrust or buoyant force and the phenomenon is called buoyancy

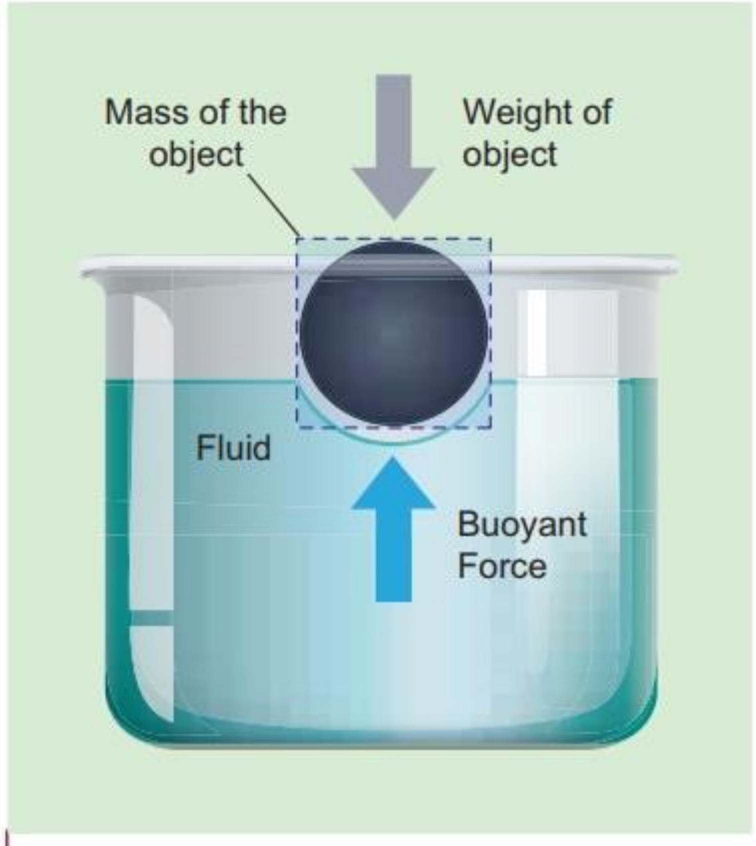
It states that when a body is partially or wholly immersed in a fluid, it experiences an upward thrust equal to the weight of the fluid displaced by it and its upthrust acts through the centre of gravity of the liquid displaced.

upthrust or buoyant force = weight of liquid displaced

and also,

U = weight of the body in air – Weight of the body in water

$= W_a - W_w$



Derivation of Fluid Pressure

Liquids exert pressure at the base of the container due to their weight. However, on the walls the pressure is same in magnitude.

Supposes a liquid in a Cylinder

Let h be the height of the liquid column in a cylinder of cross sectional area A. If ρ is the density of the liquid, then weight of the liquid column W is given by

W = mass of liquid column (m) × g = Vρg

By definition, pressure is the force acting per unit area

$$\therefore \text{Pressure} = \frac{\text{weight of liquid column}}{\text{area of cross section}} = \frac{F}{A} = \frac{W}{A}$$

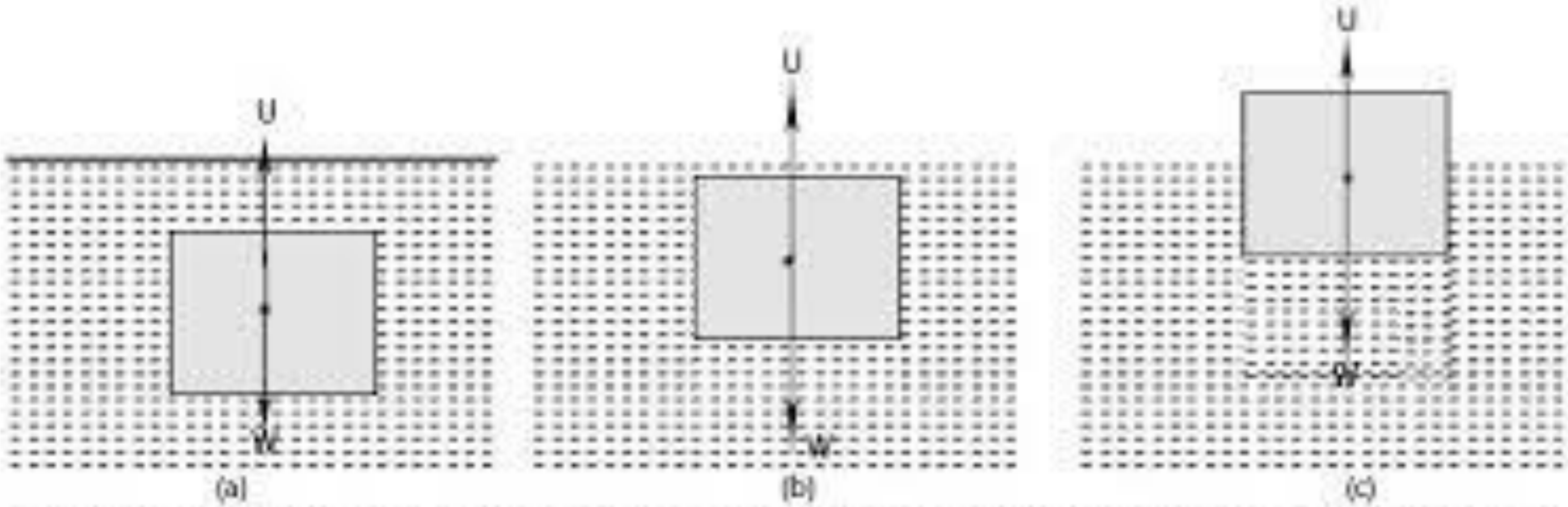
$$= \frac{Ah\rho g}{A}$$

$$= h\rho g$$

This is the expression for the pressure exerted by a liquid at a depth **h**

Principle of Floatation

When a body is immersed in a liquid, it displaces liquid equal to the volume of its immersed part. The weight of the displaced liquid provides upthrust to the body and as known as buoyant force. The weight of the liquid displaced by the body may be greater than, or equal to or less than weight of the body.



(a) A body complete inside a liquid, P>Pl. (b) A body inside a liquid, P=Pl. (c) A body floating in a liquid, P< Pl.

Consider a body of volume **V**, density **ρ** and weight **W**, let **W'** be the weight of the liquid displaced by the body.

(i) If the weight of the body is greater than the weight of displaced liquid i.e.

$W > W'$

or, $V \rho g > V \rho' g$

or $\rho > \rho'$ in this case, density of material is greater than that of the liquid and body sink.

(ii) If the weight of the body is equal to that of the displaced liquid i.e.

$W = W'$

or, $V \rho g = V \rho' g$

or, $\rho = \rho'$ in this case, the body will float but wholly immersed inside the liquid.

(iii) If the weight of the body is less than the weight of displaced liquid i.e

$W < W'$

or, $V \rho g < V \rho' g$

or $\rho < \rho'$ the body will partly the inside liquid and floats on the surface of the liquid.

Equilibrium of floating bodies

Center of Buoyancy:

- the point at which the center of gravity of the displaced liquid lie
- The centre of buoyancy is the point, through which the force of buoyancy is supposed to act. Itis always the centre of gravity of the volume of the liquid displaced. In other words, the centre of buoyancy is the centre of area of the immersed section.

Meta center:

- It is the point of intersection of the vertical line passing through C.B and original vertical line.
- It is noted that meta center exists for floating bodies only and it helps to determine whether a body is stable or not.

At equilibrium, the center of gravity C.G of the body and center of buoyancy C.B of the displaced liquid both lie on the same vertical axis.

If the floating body is slightly tilted form its equilibrium position, then the C.G and C.B will not lie on the same vertical line.

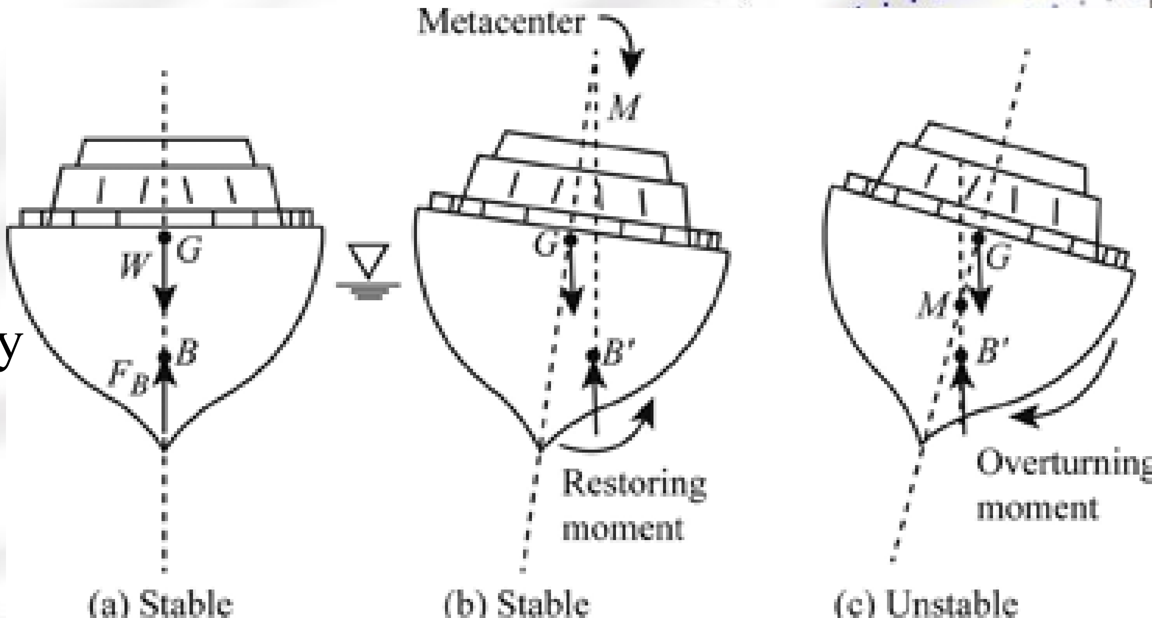
Two possible cases are there.

(a) If the C.G of the body lies below the C.B of the liquid, which is obtained in heavy bottom body then the M.C of the body will lie above the C.G of the floating body .

The body is then acted by a pair of forces, W and U acting at C.G and C.B respectively. And these two forces form a couple and brings back the body to its equilibrium position. The body restores its stable equilibrium

(b) If the C.G of the body lies above the C.B of the liquid, such as in a heavy top body and when it is tilted from the equilibrium position the M.C lies below C.G.

The couple formed by two forces W and U acting at C.G and C.B respectively rotates more the body and takes away from the equilibrium position. So, stability of the body is lost and overturn.



Examples of floatation:

1. Ships: Though the density of the materials used in ship is greater than the density of water, the structures of the ship is made such that it displaces more volume of water than it does in solid forms.
2. Iceberg: Density of ice is smaller than the density of water. So, it floats in sea with certain volume out of the water level.
3. Submarine: Submarines can float as well as sink in sea water. When water is filled up in its water tanks, the submarine becomes heavy and sinks down in sea. When water is forced out of the tanks by high air pressure through valves, the ship can float in sea.
4. Balloons: A hydrogen filled balloon is lighter than air and the air forces up it to a straight where weight of balloon is equal to the upthrust of air there.

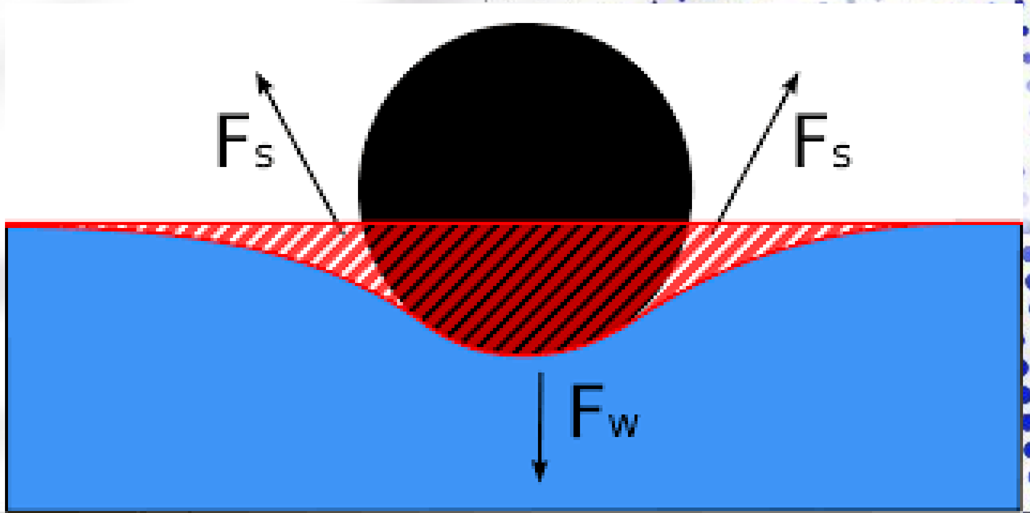
Surface Tension

- The property of liquid at rest by virtue of which its surface behaves like a stretched membrane and tries occupy minimum possible surface area.
- Mathematically, It is the force per unit length of an imaginary line drawn in the plane of the liquid surface acting at right angles to this line. i.e.

If **F** is the force acting on the imaginary line of length **l,** then

$$\text{surface tension, } T = \frac{F}{l}$$

- Its units is N m^{-1} is SI- units and dyne/cm in CGS- system.
- Dimension of surface tension $[M^1L^0T^{-2}]$



Molecular Theory of Surface

Consider two molecules P and Q as shown in Fig.. Taking them as centres and molecular range as radius, a sphere of influence is drawn around them.

- The molecule P is attracted in all directions equally by neighboring molecules. Therefore net force acting on P is zero.
- The molecule Q is on the free surface of the liquid. It experiences a net downward force because the number of molecules in the lower half of the sphere is more and the upper half is completely outside the surface of the liquid. Therefore all the molecules lying on the surface of a liquid experience only a net downward force.
- If a molecule from the interior is to be brought to the surface of the liquid, work must be done against this downward force. This work done on the molecule is stored as potential energy. *For equilibrium, a system must possess minimum potential energy.* So, the free surface will have minimum potential energy. The free surface of a liquid tends to assume minimum surface area by contracting and remains in a state of tension like a stretched elastic membrane

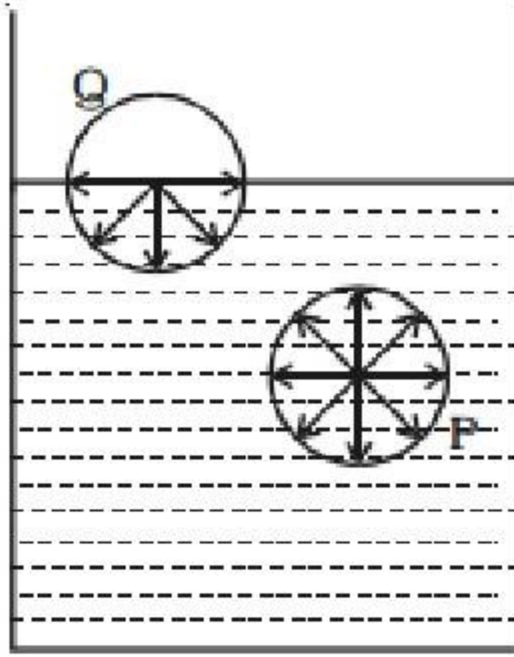


Fig. Surface tension based on molecular theory

Some Examples Explaining Surface Tension

a) Tread on a soap film
b) Floating needle
c) When a dry brush is dipped into water, its hair spread out
d) Formation of lead shots
e) Oil has less surface tension than water

Surface Energy

- The free surface of a liquid always has a tendency to contract and occupies minimum surface area.
- If surface area of a liquid increased than work-done is stored in the liquid surface as its potential energy.
- The potential energy per unit area of the surface film is called surface energy. And mathematically,

$$\sigma = \frac{\text{Work done in increasing surface area}}{\text{increase in surface area}}$$

- It is expressed in Jm^{-2} or Nm^{-1}.
- It is also called free surface energy because the mechanical work done can be released when the surface contracts.

Relation between surface tension and surface energy

Consider a rectangular frame of wire ABCD in a soap solution (Figure 7.25). Let AB be the movable wire. Suppose the frame is dipped in soap solution, soap film is formed which pulls the wire AB inward due to surface tension. Let *F* be the force due to surface tension, then

$F = (2T)l$

here, 2 is introduced because it has two free surfaces. Suppose AB is moved by a small distance Δx to new a position $A'B'$. Since the area increases, some work has to be done against the inward force due to surface tension.

Work done = Force × distance = $(2T\ l)\ (\Delta x)$

Increase in area of the film $\Delta A = (2l)$

$(\Delta x) = 2l\ \ x$

Therefore, Surface energy $\sigma = \dfrac{\text{Work done in increasing surface area}}{\text{increase in surface area}}$

$$= \frac{2Tl\Delta x}{2l\Delta x} = T$$

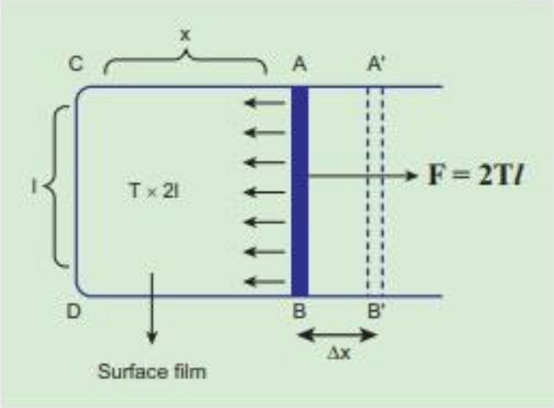


Figure 7.25 A horizontal soap film on a rectangular frame of wire ABCD

Excess pressure on curved surface of a liquid

- The free surface of a liquid becomes curved when it has contact with a solid. Depending upon the nature of liquid-air or liquid-gas interface, the magnitude of interfacial surface tension varies.

- In other words, as a consequence of surface tension, the above such interfaces have energy and for a given volume, the surface will have a minimum energy with least area. Due to this reason, the liquid drop becomes spherical (for a smaller radius).

- When the free surface of the liquid is curved, there is a difference in pressure between the inner and outer the side of the surface

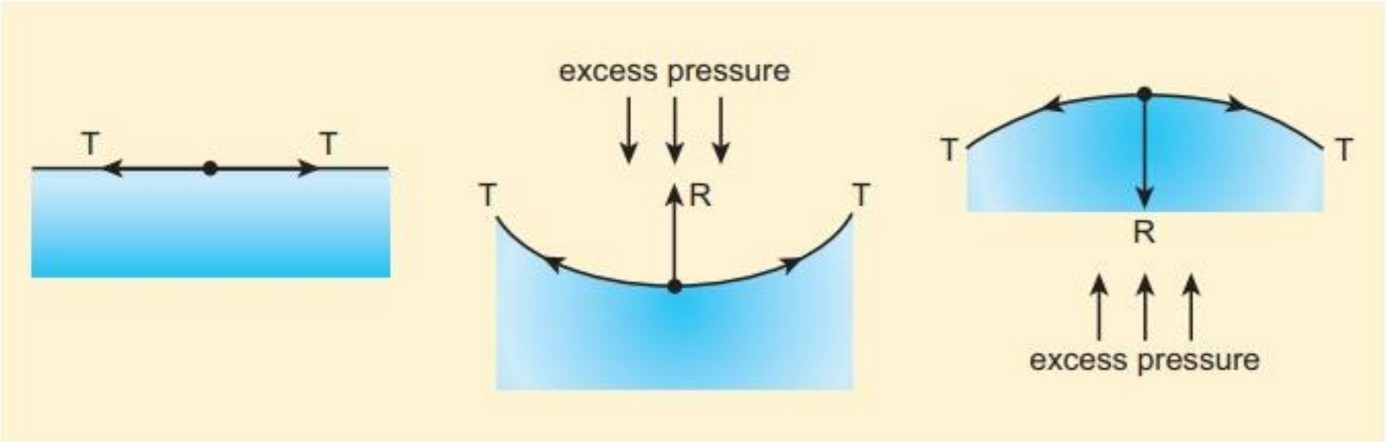
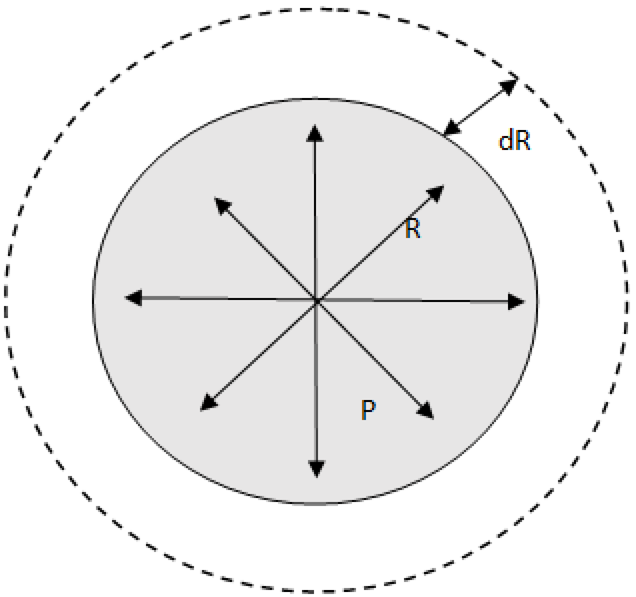


Figure7.27 Excess of pressure across a liquid surface

i) When the liquid surface is plane, the forces due to surface tension (T, T) act tangentially to the liquid surface in opposite directions. Hence, the resultant force on the molecule is zero. Therefore, in the case of a plane liquid surface, the pressure on the liquid side is equal to the pressure on the vapour side

ii) When the liquid surface is curved, every molecule on the liquid surface experiences forces (F_T, F_T) due to surface tension along the tangent to the surface. Resolving these forces into rectangular components, we find that horizontal components cancel out each other while vertical components get added up. Therefore, the resultant force normal to the surface acts on the curved surface of the liquid. Similarly, for a convex surface, the resultant force is directed inwards towards the centre of curvature, whereas the resultant force is directed outwards from the centre of curvature for a concave surface. Thus, for a curved liquid surface in equilibrium, the pressure on its concave side is greater than the pressure on its convex side.

Excess pressure inside a liquid Drop



Consider a drop of liquid R as shown in the figure. The molecules lying on the surface of the liquid drop due to surface tension will experience a resultant force acting inwards perpendicular to the surface. As a result, the pressure inside the drop must be greater than the pressure outside it. The excess pressure inside the drop will provide a force acting outwards perpendicular to the surface, to balance the resultant force due to surface tension.

Let T be the surface tension and P be the excess pressure inside the drop. Suppose due to excess pressure, there be an increase in the radius of the drop by quantity dR. In such case we can write, work done by excess pressure,

W = Force × displacement = (Excess pressure × area) × displacement

or, $W = P \times 4\pi R^2 \times dR$ …….. (i)

Increase in surface area of the drop = Final surface area – Initial surface area

$= 4\pi (R + dR)^2 - 4\pi R^2$

$= 4\pi [R^2 + 2. R. dR + dR^2] - 4\pi R^2$

$= 8\pi R\, dR$ [since dR^2 is very small, it is neglected]

Increase in surface energy = Increase in surface area × surface tension

$= 8\pi R\, dR \times T$

From equation (i) and (ii), we get

$P \times 4\pi R^2 \times dR = 8\pi R\, dR \times T$

$\therefore P = \frac{2T}{R}$

or, $P_{in} - P_{out} = \frac{2T}{R}$

Excess Pressure of Air Bubble:

Since the air bubble has only one free surface, so the excess pressure inside is given by $P = \frac{2T}{R}$

or, $P_{in} - P_{out} = \frac{2T}{R}$

Excess Pressure inside liquid Bubble or soap Bubble :

Since a liquid bubble has two free surfaces, so the excess pressure is given by $P = 2 \times \frac{2T}{R}$

or, $P_{in} - P_{out} = \frac{4T}{R}$

Shape of Liquid Surface

Refer to your text book…

Angle of contact

- When the free surface of a liquid comes in contact with a solid, it becomes curved at the point of contact. The angle between the tangent to the liquid surface at the point of contact of the liquid with the solid and the solid surface inside the liquid is called angle of contact.
- In Fig. , QR is the tangent drawn at the point of contact Q. The angle PQR is called the angle of contact. When a liquid has concave meniscus, the angle of contact is acute. When it has a convex meniscus, the angle of contact is obtuse.

The angle of contact depends on the nature of liquid and solid in contact. For water and glass, θ lies between 8^o and 18^o. For pure water and clean glass, it is very small and hence it is taken as zero. The angle of contact of mercury with glass is 138^o

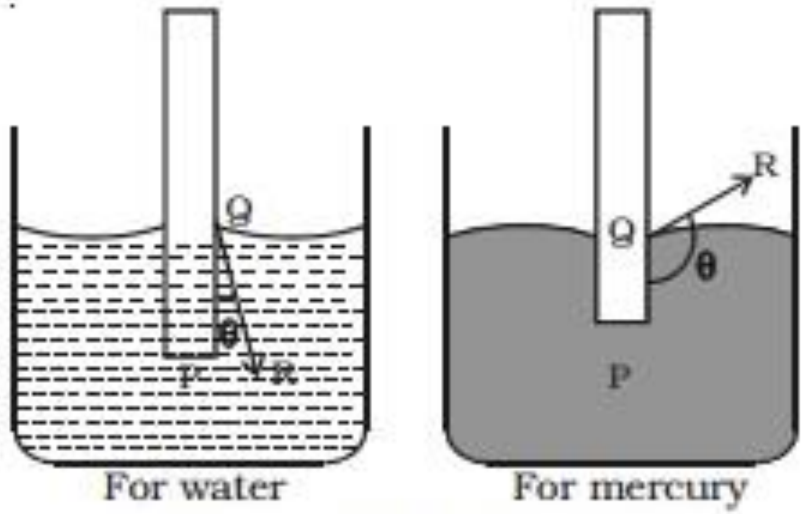


Fig. Angle of contact

Capillarity

- The property of surface tension gives rise to an interesting phenomenon called capillarity.
- When a capillary tube is dipped in water, the water rises up in the tube. The level of water in the tube is above the free surface of water in the beaker (capillary rise).
- When a capillary tube is dipped in mercury, mercury also rises in the tube. But the level of mercury is depressed below the free surface of mercury in the beaker (capillary fall).
- The rise of a liquid in a capillary tube is known as capillary

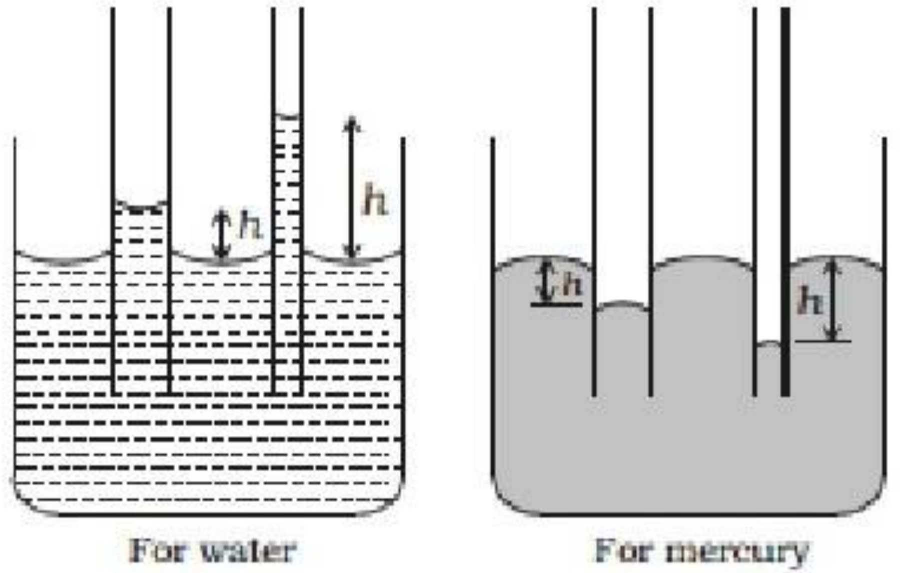


Fig. Capillary rise

Illustrations of Capillarity:

1. A blotting paper absorbs ink by capillary action. The pores in the blotting paper act as capillaries.
2. The oil in a lamp rises up the wick through the narrow spaces between the threads of the wick.
3. A sponge retains water due to capillary action.
4. Walls get damped in rainy season due to absorption of water by bricks .

Measurement of surface Tension by Capillary Rise Method

Let us consider a capillary tube of uniform bore dipped vertically in a beaker containing water. Due to surface tension, water rises to a height h in the capillary tube as shown in Fig.. The surface tension T of the water acts inwards and the reaction of the tube R outwards. R is equal to T in magnitude but opposite in direction. This reaction R can be resolved into two rectangular components.

(i) Horizontal component $R \sin \theta$ acting radially outwards

(ii) Vertical component $R \cos \theta$ acting upwards.

The horizontal component acting all along the circumference of the tube cancel each other whereas the vertical component balances the weight of water column in the tube.

The surface tension force F_T, acts along the tangent at the point of contact downwards and its reaction force upwards. Surface tension T, is resolved into two components i) Horizontal component T sinθ and ii) Vertical component T cosθ acting upwards, all along the whole circumference of the meniscus

Total upward force = R cos θ × circumference of the tube

(i.e) $F = 2\pi r\ R \cos\theta$ or

$$F = 2\pi r\ T \cos\theta \ \ldots\ldots\ldots\ldots(1)$$

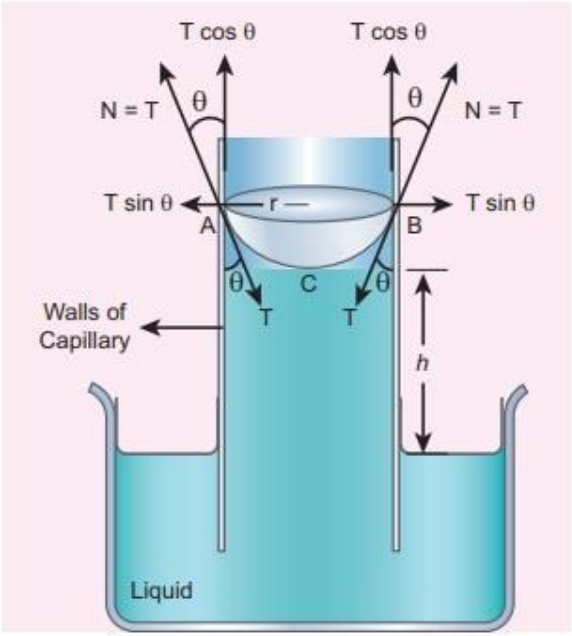


Figure 7.31 Capillary rise by surface tension

where θ is the angle of contact, *r* is the radius of the tube. Let ρ be the density of water and *h* be the height to which the liquid rises inside the tube. Then

$$\begin{pmatrix}\text{the volume of}\\ \text{liquid column in}\\ \text{the tube, V}\end{pmatrix} = \begin{pmatrix}\text{volume of the}\\ \text{liquid column of radius r}\\ \text{height h}\end{pmatrix}$$

$$+\begin{pmatrix}\text{volume of liquid of radius r}\\ \text{and height r - Volume of the}\\ \text{hemisphere of radius r}\end{pmatrix}$$

$$V = \pi r^2 h + \left(\pi r^2 \times r - \frac{2}{3}\pi r^3\right) \Rightarrow V = \pi r^2 h + \frac{1}{3}\pi r^3$$

The upward force supports the weight of the liquid column above the free surface, therefore

$$2\pi r T\cos\theta = \pi r^2\left(h + \frac{1}{3}r\right)\rho g \Rightarrow T = \frac{r\left(h + \frac{1}{3}r\right)\rho g}{2\cos\theta}$$

If the capillary is a very fine tube of radius (i.e., radius is very small) then r/3 can be neglected when it is compared to the height h. Therefore,

$$T = \frac{r\rho g h}{2\cos\theta}$$

Liquid rises through a height h,

$$h = \frac{2\,T\cos\theta}{r\rho g} \qquad h \; \alpha \; \frac{1}{r}$$

This implies that the capillary rise (h) is inversely proportional to the radius (r) of the tube. i.e, the smaller the radius of the tube greater will be the capillarity

This expression is called ascent formula. In this case angle of contact is acute.

- Narrower the tube, the greater is the height to which the liquid rises.
- If the tube is held vertically in a liquid that has a convex meniscus, then the angle of contact is obtuse so that cos θ is negative and hence liquid will suffer capillary depression.

Factors affecting surface tension

- Impurities present in a liquid appreciably affect surface tension. A highly soluble substance like salt increases the surface tension whereas sparingly soluble substances like soap decreases the surface tension.
- The surface tension decreases with rise in temperature. The temperature at which the surface tension of a liquid becomes zero is called critical temperature of the liquid.

Application of Surface tension

- During stormy weather, oil is poured into the sea around the ship. As the surface tension of oil is less than that of water, it spreads on water surface. Due to the decrease in surface tension, the velocity of the waves decreases. This reduces the wrath of the waves on the ship.
- Lubricating oils spread easily to all parts because of their low surface tension.

- Dirty clothes cannot be washed with water unless some detergent is added to water. When detergent is added to water, one end of the hairpin shaped molecules of the detergent get attracted to water and the other end, to molecules of the dirt. Thus the dirt is suspended surrounded by detergent molecules and this can be easily removed. This detergent action is due to the reduction of surface tension of water when soap or detergent is added to water.
- Cotton dresses are preferred in summer because cotton dresses have fine pores which act as capillaries for the sweat.

Newton's Law of Viscosity, Coefficient of Viscosity

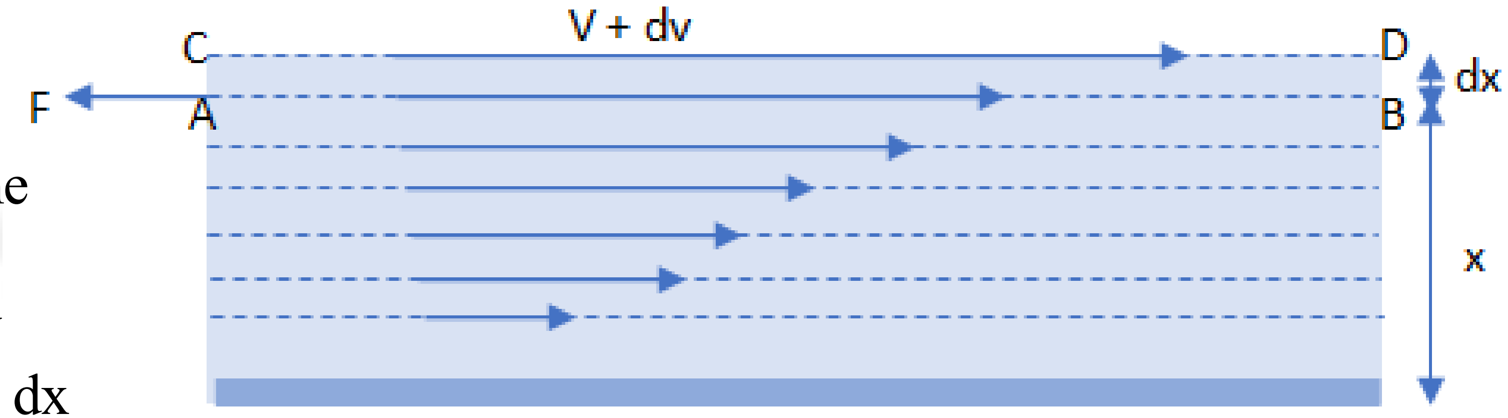
The property of a liquid or gas by virtue of which an internal friction comes in to play when the fluid is in motion and oppose the relative motion of its different layers is called Viscosity

Consider a liquid flowing steadily over a fixed solid horizontal surface as shown in the figure. Every layer of liquid moves parallel to the fixed surface. The layer in contact with the fixed surface is at rest while the velocity of other layers increases uniformly upwards. Consider two layers AB and CD moving with velocity v and v + dv at a distance x and x + dx respectively from the fixed solid surface.

The change of velocity divided by the distance, dv/dx, in a direction perpendicular to the velocity is called the velocity gradient.

According to newton, the viscous force F depends upon the following factors:

- It is directly proportional to area A or the layers in contact. i.e; F α A
- It is directly proportional to the velocity gradient between the layers. i.e; F α dv/ dx



Combining these two factors, we have

$F \alpha A\, dv/ dx$

$F = - \eta A\, dv/ dx$ …... (i)

Where η is the constant of proportionality called coefficient of viscosity. Its value depends upon the nature of the liquid. If $A = 1$ and $dv/ dx = 1$.

Then, from equation (i), we have

$\eta = -F$

Hence, the coefficient of viscosity of a liquid is defined as the viscous drag or viscous force acting per unit area of the layer having unit velocity gradient perpendicular to the direction of the flow of the liquid.

Units and dimensional formula of coefficient of Viscosity (η)

- The coefficient of viscosity of a fluid is $\eta = \frac{F}{A\frac{dv}{dx}}$

$$1 \text{ poise} = \frac{1 \text{ dyne}}{1\text{cm}^2 \frac{1 \text{ cm} S^{-1}}{\text{cm}}} = 1 \text{ dyne cm}^{-2} \text{ sec}$$

$$\therefore 1 \text{ poise} = 1 \text{ dyne cm}^{-2} \text{ sec}$$

$$1 \text{ poise} = 10^{-5}\text{N} \times 1 \text{ sec} \times 10^{4}\text{m}^{-2} = 0.1\text{N s m}^{-2}$$

$$1 \text{ deca-poise} = 10 \text{ poise} = 1\text{N s m}^{-2}$$

And dimension,

$$\eta = \frac{[MLT^{-2}]}{[L^2][\frac{[LT^2]}{L}]} = [ML^{-1}T^{-1}]$$

in SI –units $kgm^{-1}s^{-1}$

Stream- line and turbulent flow

If every particle of the liquid follows exactly the path of its preceding particle and has the same velocity of its preceding particle at every point. Such type of flow of liquid is called stream line flow.

Let **abc** be the path of flow of a liquid and v_1, v_2 and v_3 be the velocities of the liquid at the points a, b and c respectively. During a streamline flow, all the particles arriving at 'a' will have the same velocity v_1 which is directed along the tangent at the point 'a'. A particle arriving at b will always have the same velocity v_2. This velocity v_2 may or may not be equal to v_1. Similarly all the particles arriving at the point c will always have the same velocity v_3. In other words, in the streamline flow of a liquid, the velocity of every particle crossing a particular point is the same.

The streamline flow is possible only as long as the velocity of the fluid does not exceed a certain value. This limiting value of velocity is called critical velocity.

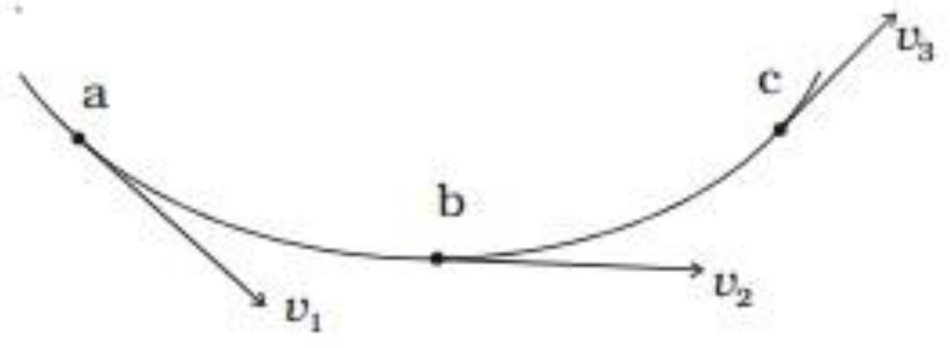


Fig. Steamline flow

Laminar Flow :

If a liquid is flowing over a horizontal surface with a steady flow and moves in the form of layers of different velocities which donot mix with each other then the flow of liquid is called laminar flow.

Turbulent flow:

- When the velocity of a liquid exceeds the critical velocity, the path and velocities of the liquid become disorderly. At this stage, the flow loses all its orderliness and is called turbulent flow. Some examples of turbulent flow are :

a. After rising a short distance, the smooth column of smoke from an incense stick breaks up into irregular and random patterns.
b. The flash - flood after a heavy rain.

Critical velocity and Reynolds Number

When the fluid flows through a tube with small velocities, motion will be stream line but on increasing the velocity when it becomes greater than a certain limiting value, the motion becomes turbulent. This is called critical velocity.

the velocity v_c depends upon (i) coefficient of viscosity of fluid (ii) density ρ of the fluid and (iii) lateral dimensions r of the tube in which liquid is flowing and

Let $v_c \alpha \eta^a \rho^b r^c$

$v_c = k \eta^a \rho^b r^c$ ....(1)

where k is dimensionless constant.

Rewriting equation (1) in terms of dimensions,

$[LT^{-1}] = [ML^{-1} T^{-1}]^{a} [ML^{-3}]^{b}[L]^{c}$

$[LT^{-1}] = [M]^{a+b} [L]^{-a+3b+c}[T]^{-a}$

Equating the powers of L, M and T on both sides and fromthe principle of homogeneity we get a = -1, b = -1 and c = -1

Substituting in equation (1),

$v_c = k\frac{\eta}{\rho r}$ this expression for critical velocity is called Reynolds formula and the constant k is Reynolds number.

$k = \frac{v_c \rho r}{\eta}$; in order to maintain the streamline flow the value of k must small i.e. critical velocity, density of fluid and radius of the tube must be small and the coefficient of viscosity must be large.

Poiseuille's Formula

Poiseuille investigated the steady flow of a liquid through a capillary tube. He derived an expression for the volume of the liquid flowing per second through the tube.

- Directly proportional to the difference of pressure, P between the two ends of the tube

$$V \propto P$$

- Directly proportional to the fourth power of radius, r of the capillary tube

$$V \propto r^4$$

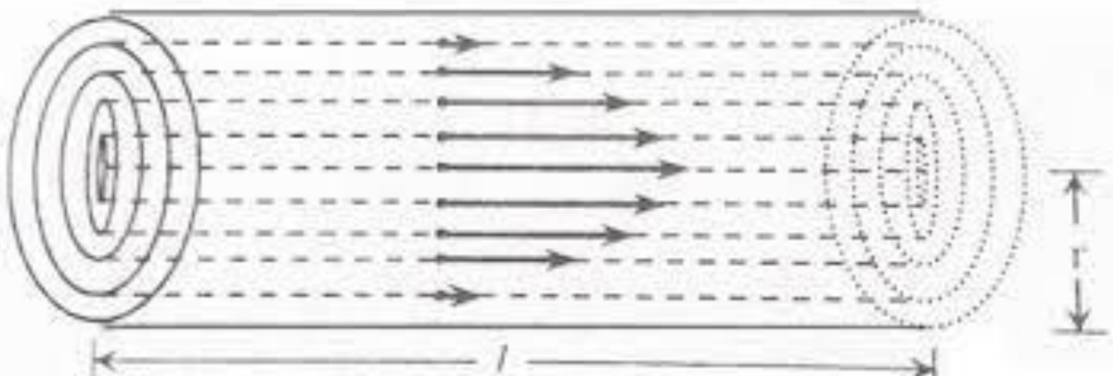
- inversely proportional to the coefficient of viscosity, η of the liquid

$$V \propto 1/\eta$$

- inversely proportional to the length , l of the capillary tu

$$V \propto 1/l$$

And combining these factors $V = \frac{\pi P r^4}{8\eta l}$



Stream line flow of a liquid in a capillary tube

Poiseuille's Formula

Poiseuille investigated the steady flow of a liquid through a capillary tube. He derived an expression for the volume of the liquid flowing per second through the tube.

Consider a liquid of co-efficient of viscosity η flowing, steadily through a horizontal capillary tube of length l and radius r. If P is the pressure difference across the ends of the tube, then the volume V of the liquid flowing per second through the tube depends on η, r and the pressure gradient p/l.

(i.e) $V \alpha \eta^a r^b (P/l)^c$

$V = k \eta^a r^b (P/l)^c$……..(1)

where k is a constant of proportionality. Rewriting equation (1) in terms of dimensions,

The dimension of V = $[L^3T^{-1}]$

The dimension of P/l = $[ML^{-2} T^{-2}]$

The dimension of η = $[ML^{-1} T^{-1}]$

The dimension of r = [L]

Putting the dimensions of the quantities in equation (1)

$[L^3T^{-1}] = [ML^{-1} T^{-1}]^a [L]^b [ML^{-1}T^{-2} / L]^c$

Equating the powers of L, M and T on both sides we get a = -1, b = 4 and c = 1

Substituting in equation (1),

$$V = k\frac{Pr^4}{\eta l}$$

Experimentally k was found to be equal to π/ 8

$$V = \frac{\pi Pr^4}{8\eta l}$$

This is known as Poiseuille's equation

Applications of Poiseuille's Formula:

- The Poiseuille's formula is used to study the fluid feeding by insects that are sucking through mouthparts.
- It is used to determine the blood flow through the veins in the body.
- By using the poiseuille's formula, it is possible to describe the mineral melt motion in mineral fibre production.
- It is applied to the flow of liquid through the drinking straw.

Stokes' Law

When a body falls through a highly viscous liquid, it drags the layer of the liquid immediately in contact with it. This results in a relative motion between the different layers of the liquid. As a result of this, the falling body experiences a viscous force F. Stoke performed many experiments on the motion of small spherical bodies in different fluids and concluded that the viscous force F acting on the spherical body depends on

(i) Coefficient of viscosity η of the liquid

(ii) Radius a of the sphere and

(iii) Velocity v of the spherical body. Dimensionally it can be proved that

$F = k\,\eta a v$

Experimentally Stoke found that

$k = 6\pi$

$F = 6\pi\,\eta a v$

This is Stokes' law.

Derivation of Stokes' Law by Dimensional Analysis

Do yourself (refer from your text book)

Expression for terminal velocity

Consider a metallic sphere of radius 'a' and density ρ to fall under gravity in a liquid of density σ. The viscous force F acting on the metallic sphere increases as its velocity increases. A stage is reached when the weight W of the sphere becomes equal to the sum of the upward viscous force F and the upward thrust U due to buoyancy (Fig.). Now, there is no net force acting on the sphere and it moves down with a constant velocity v called terminal velocity.

$W - F - U = O$ ...(1)

Terminal velocity of a body is defined as the constant velocity acquired by a body while falling through a viscous liquid.

From (1), $W = F + U$ .. ...(2)

According to Stoke's law, the viscous force F is given by $F = 6\pi\eta av$.

The buoyant force U = Weight of liquid displaced by the sphere

$= \frac{4}{3}\pi a^3 \sigma g$

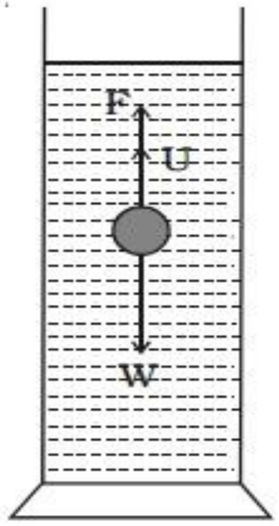


Fig. Sphere falling in a viscous liquid

The weight of the sphere $W = \frac{4}{3}\pi a^3 \rho g$

Substituting in equation (2)

$$\frac{4}{3}\pi a\,\rho g = 6\pi\,\eta a v + \frac{4}{3}\pi a^3\,\sigma\,g$$

$$\therefore v = \frac{2\,a^2(\rho - \sigma)g}{9\eta}$$

Application of Stoke's law

- Falling of rain drops: When the water drops are small in size, their terminal velocities are small. Therefore they remain suspended in air in the form of clouds. But as the drops combine and grow in size, their terminal velocities increases because $v \propto a^2$. Hence they start falling as rain.

Equation of continuity

In order to discuss the mass flow rate through a pipe, it is necessary to assume that the flow of fluid is steady, the flow of the fluid is said to be steady if at any given point, the velocity of each passing fluid particle remains constant with respect to time. Under this condition, the path taken by the fluid particle is a streamline.

- Consider a pipe AB of varying cross sectional area a_1 and a_2 such that $a_1 > a_2$. A non-viscous and incompressible liquid flows steadily through the pipe, with velocities v_1 and v_2 in area a_1 anda_2, respectively as shown in Figure 7.32.

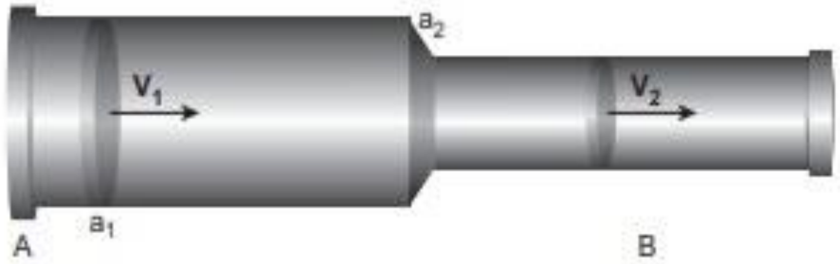


Fig 7.32 A streamlined flow of fluid through a pipe of varying cross sectional area

Let m_1 be the mass of fluid flowing through section A in time Δt, $m_1 = (a_1 v_1 \; t)\;\rho$
Let m_2 be the mass of fluid flowing through section B in time t, $m_2 = (a_2 v_2 \; t)\;\rho$
For an incompressible liquid, mass is conserved $m_1 = m_2$

$$a_1 v_1 \; \Delta t \; \rho = a_2 v_2 \; \Delta t \; \rho$$

$$a_1 v_1 = a_2 v_2 \longrightarrow \quad av = \text{constant}$$

which is called the equation of continuity and it is a statement of conservation of mass in the flow of fluids.

In general, $a\,v$ = constant, which means that the volume flux or flow rate remains constant throughout the pipe. In other words, the smaller the cross section, greater will be the velocity of the fluid.

Application

- The common applications of continuity are used in pipes, tubes and ducts with the following fluids or gases, rivers, canals etc.
- It is used in Bernoulli's theorem

Total energy of a liquid

A liquid in motion possesses pressure energy, kinetic energy and potential energy

(i) Pressure energy

It is the energy possessed by a liquid by virtue of its pressure. Consider a liquid of density ρ contained in a wide tank T having a side tube near the bottom of the tank as shown in Fig.

A frictionless piston of cross sectional area 'A' is fitted to the side tube. Pressure exerted by the liquid on the piston is $P = h \rho g$ where h is the height of liquid column above the axis of the side tube.

If x is the distance through which the piston is pushed inwards, then

Volume of liquid pushed into the tank = Ax

∴ Mass of the liquid pushed into the tank = Ax ρ

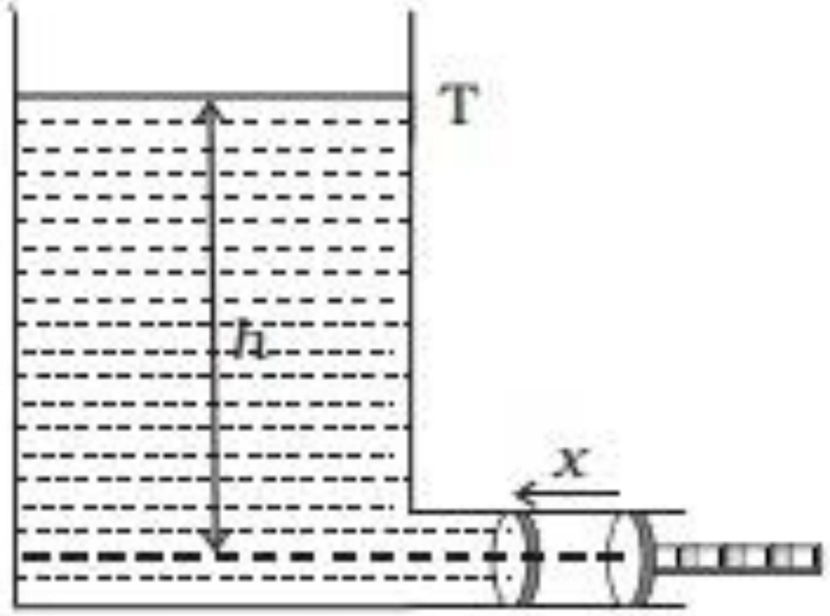


Fig. Pressure energy

As the tank is wide enough and a very small amount of liquid is pushed inside the tank, the height h and hence the pressure P may be considered as constant.
Work done in pushing the piston through the distance x = Force on the piston × distance moved
(i.e) W = PAx
This work done is the pressure energy of the liquid of mass Axρ.
∴ Pressure energy per unit mass of the liquid = PAx /Ax ρ = P/ ρ

(ii) Kinetic energy

It is the energy possessed by a liquid by virtue of its motion.

If m is the mass of the liquid moving with a velocity v, the kinetic energy of the liquid = $\frac{1}{2}mv^2$

Kinetic energy per unit mass = $\frac{1}{2}mv^2$ / m = $\frac{v^2}{2}$

(iii) Potential energy

- It is the energy possessed by a liquid by virtue of its height above the ground level.

If *m* is the mass of the liquid at a height *h* from the ground level, the potential energy of the liquid = *mgh*

Potential energy per unit mass = $\frac{mgh}{m} = gh$

Total energy of the liquid in motion = pressure energy + kinetic energy + potential energy.

Total energy per unit mass of the flowing liquid = $\frac{P}{\rho} + \frac{v^2}{2} + gh$

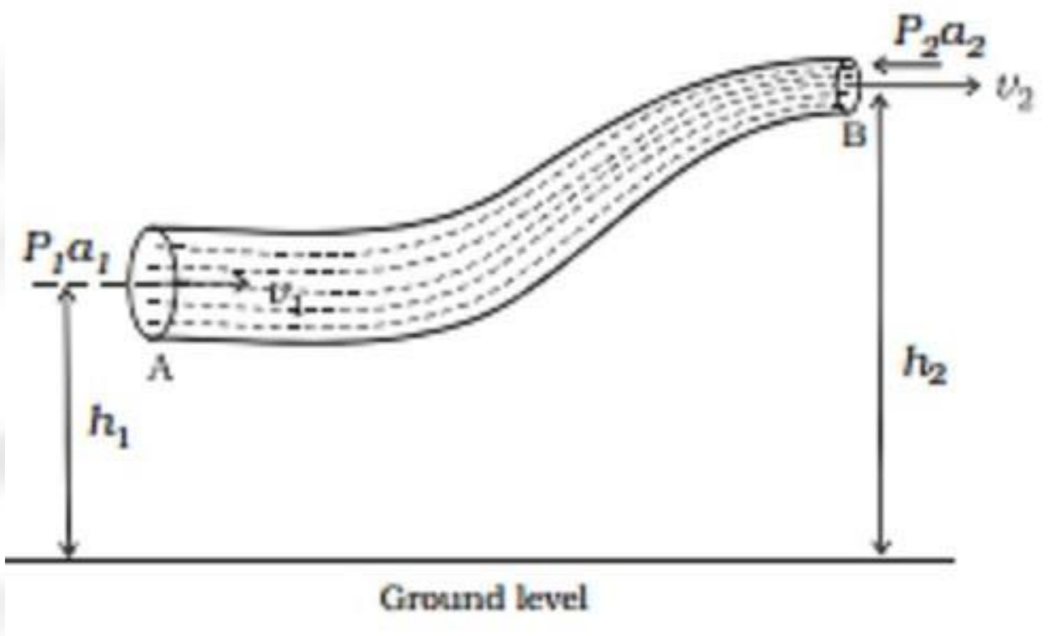
Bernoulli's Theorem

In 1738, Daniel Bernoulli proposed a theorem for the streamline flow of a liquid based on the law of conservation of energy. According to Bernoulli's theorem, for the streamline flow of a non-viscous and incompressible liquid, the sum of the pressure energy, kinetic energy and potential energy per unit mass is a constant.

$$\frac{P}{\rho} + \frac{v^2}{2} + gh = \text{constant.}$$

This equation is known as Bernoulli's equation.

Consider streamline flow of a liquid of density ρ through a pipe AB of varying cross section. Let P_1 and P_2 be the pressures and a_1 and a_2, the cross sectional areas at A and B respectively. The liquid enters A normally with a velocity v_1 and leaves B normally with a velocity v_2.



The liquid is accelerated against the force of gravity while flowing from A to B, because the height of B is greater than that of A from the ground level. Therefore P_1 is greater than P_2. This is maintained by an external force

The mass m of the liquid crossing per second through any section of the tube in accordance with the equation of continuity is

$a_1 v_1 \rho = a_2 v_2 \rho = m$

or

$a_1 v_1 = a_2 v_2 = m/\rho = V$

As $a_1 > a_2$, $v_1 < v_2$

The force acting on the liquid at A = $P_1 a_1$

The force acting on the liquid at B = $P_2 a_2$

Work done per second on the liquid at A = $P_1 a_1 \times v_1 = P_1 V$

Work done by the liquid at B = $P_2 a_2 \times v_2 = P_2 V$

∴ Net work done per second on the liquid by the pressure energy

in moving the liquid from A to B is = $P_1 V - P_2 V$ ...(2)

If the mass of the liquid flowing in one second from A to B is m, then increase in potential energy per second of liquid from A to B is $mgh_2 - mgh_1$

Increase in kinetic energy per second of the liquid

$$= \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

According to work-energy principle,

work done per second by the pressure energy = Increase in potential energy per second + Increase in kinetic energy per second

$$\frac{P}{\rho} + \frac{v^2}{2} + gh = \text{constant} \quad \ldots\ldots\ldots 3$$

This is Bernoulli's equation. Thus the total energy of unit mass of liquid remains constant.

Dividing equation (3) by g,

$$\frac{P}{\rho g} + \frac{v^2}{2g} + h = \text{constant}$$

Each term in this equation has the dimension of length and hence is called head. $\frac{P}{\rho g}$ is called pressure head, $\frac{v^2}{2g}$ is velocity head and h is the gravitational head.

Special cases :

If the liquid flows through a horizontal tube, $h_1 = h_2$. Therefore there is no increase in potential energy of the liquid i.e. the gravitational head becomes zero.

equation (3) becomes

$$\frac{P}{\rho} + \frac{v^2}{2} = \text{a constant}$$

This is another form of Bernoulli's equation

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Application of Bernoulli's Theorem

(a) Blowing off roofs during wind storm :

During cyclonic condition, the roof is blown off without damaging the other parts of the house. In accordance with the Bernoulli's principle, the high wind blowing over the roof creates a low-pressure P_1 . The pressure under the roof P_2 is greater. Therefore, this pressure difference ($P_2 - P_1$) creates an up thrust and the roof is blown off.

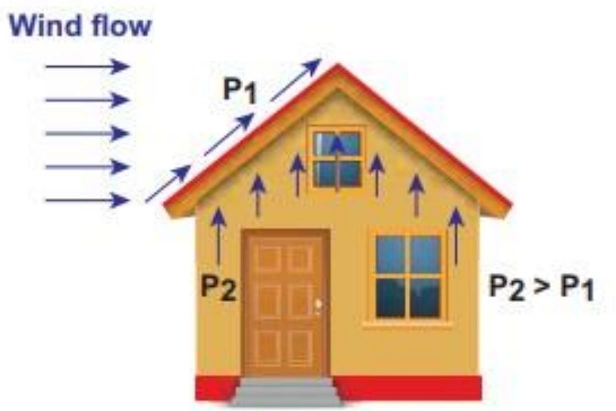

Figure 7.34 Roofs of the huts or houses

(b) Aerofoil lift :

The wing of an airplane (aerofoil) are so designed that its upper surface is more curved than the lower surface and the front edge is broader than the real edge. As the aircraft moves, the air moves faster above the aerofoil than the bottom a shown in fig.

According to Bernoulli's Principle, the pressure of air below is greater than above, which creates an upthrust called the dynamic lift to the aircraft/

Figure 7.35 Aerofoil lift

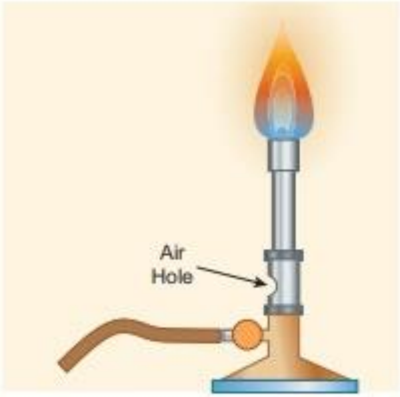

Figure 7.36 Bunsen burner

(c) Bunsen Burner:

In this, the gas comes out of the nozzle with high velocity, hence the pressure in the stem decreases. So outside air reaches into the burner through an air vent and the mixture of air and gas gives a blue flame as shown in Figure

(d) Venturi- Meter

- This device is used to measure the rate of flow (or say flow speed) of the incompressible fluid flowing through a pipe. It works on the principle of Bernoulli's theorem.
- It consists of two wider tubes A and A' (with cross sectional area A) connected by a narrow tube B (with cross sectional area a).
- A manometer in the form of U-tube is also attached between the wide and narrow tubes as shown in Figure7.37. The manometer contains a liquid of density 'ρ_m'.

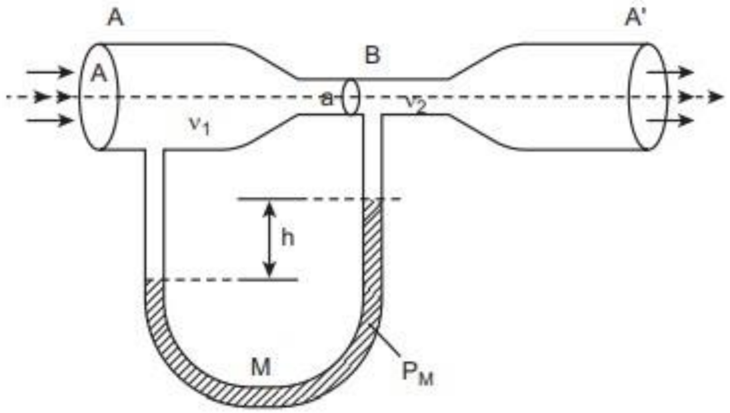

Figure 7.37 A schematic diagram of venturimeter

- Let P_1 be the pressure of the fluid at the wider region of the tube A. Let us assume that the fluid of density 'ρ' flows from the pipe with speed 'v_1' and into the narrow region, its speed increases to 'v_2'. According to the Bernoulli's equation, this increase in speed is accompanied by a decrease in the fluid pressure P_2 at the narrow region of the tube B. Therefore, the pressure difference between the tubes A and B is noted by measuring the height difference ($\Delta P = P_1 - P_2$) between the surfaces of the manometer liquid

From the equation of continuity, we can say that $Av_1 = av_2$ and $v_2 = \frac{A}{a} v_1$

Using 's Bernoulli's equation

$$P_1 + \rho \frac{v_1^2}{2} = P_2 + \rho \frac{v_2^2}{2} = P_2 + \rho \frac{1}{2}\left(\frac{A}{a} v_1\right)^2$$

From the above equation, the pressure difference $\Delta P = P_1 - P_2 = \rho \frac{v_1^2}{2} \frac{(A^2 - a^2)}{a^2}$

Thus, the speed of flow of fluid at the wide end of the tube A

$$v_1^2 = \frac{2(\Delta P)a^2}{\rho(A^2 - a^2)} \Rightarrow v_1 = \sqrt{\frac{2(\Delta P)a^2}{\rho(A^2 - a^2)}}$$

The volume of the liquid out per second is

$$V = Av_1 = A\sqrt{\frac{2(\Delta P)a^2}{p(A^2 - a^2)}} = aA\sqrt{\frac{2(\Delta P)}{p(A^2 - a^2)}}$$

Or, V $= Aa\sqrt{\frac{2gh}{A^2 - a^2}}$ where, $\Delta P = \rho g h$

Solved Problems

1. Two pistons of a hydraulic lift have diameters of 60 cm and 5 cm. What is the force exerted by the larg piston when 50 N is placed on the smaller piston?

Solution

Since, the diameter of the pistons are given, we can calculate the radius of the piston

$$r = \frac{D}{2}$$

$$\text{Area of smaller piston, } A_1 = \pi\left(\frac{5}{2}\right)^2 = \pi(2.5)^2$$

$$\text{Area of larger piston, } A_2 = \pi\left(\frac{60}{2}\right)^2 = \pi(30)^2$$

$$F_2 = \frac{A_2}{A_1} \times F_1 = (50N) \times \left(\frac{30}{2.5}\right)^2 = 7200\,N$$

This means, with the force of 50 N, the force of 7200 N can be lifted

2. Let 2 .4×10^{-4} J of work is done to increase the area of a film of soap bubble from 50 cm2 to 100 cm^2. Calculate the value of surface tension of soap solution.

Solution:

A soap bubble has two free surfaces, therefore increase in surface area

$\Delta A = A_2 - A_1 = 2(100\text{-}50) \times 10^{-4} m^2 = 100 \times 10^{-4} m^2$.

Since, work done $W = T \times \Delta A \Rightarrow T =$

$$\frac{W}{\Delta A} = \frac{2.4\times10^{-4} J}{100\times10^{-4} m^2} = 2.4\times10^{-2} N m^{-1}$$

3. If excess pressure is balanced by a column of oil (with specific gravity 0.8) 4 mm high, where R 2.0 cm, find the surface tension of the soap bubble.

Solution

The excess of pressure inside the soap bubble is

$$\Delta P = P_2 - P_1 = \frac{4T}{R}$$

But $\Delta P = P_2 - P_1 = \rho g h \Rightarrow \rho g h = \frac{4T}{R}$

$\Rightarrow$ Surface tension,

$$T = \frac{\rho g h R}{4} = \frac{(800)(9.8)(4\times10^{-3})(2\times10^{-2})}{4} =$$

$$T = 15.68\times10^{-2} N m^{-1}$$

4. Mercury has an angle of contact equal to 140° with soda lime glass. A narrow tube of radius 2 *mm*, made of this glass is dipped in a trough containing mercury. By what amount does the mercury dip down in the tube relative to the liquid surface outside?. Surface tension of mercury T=0.456 Nm^{-1}; Density of mercury ρ = 13.6 × 103 *kg m*-3

Solution,

Capillary desent,

$$h = \frac{2T\cos\theta}{r\rho g} = \frac{2\times(0.465\,Nm^{-1})(\cos 140^0)}{(2\times10^{-3}m)(13.6\times10^3)(9.8\,ms^{-2})}$$

$$\Rightarrow h = -6.89\times10^{-4}\,m$$

Where, negative sign indicates that there is fall of mercury (mercury is depressed) in glass tube.

5. In a normal adult, the average speed of the blood through the aorta (radius $r = 0.8$ *cm*) is 0.33 *ms*-1. From the aorta, the blood goes into major arteries, which are 30 in number, each of radius 0.4 *cm*. Calculate the speed of the blood through the arteries.

Solution:

$a_1 v_1 = 30\, a_2 v_2 \quad \Rightarrow \pi\, r_1^2 v_1 = 30\, \pi\, r_2^2 v_2$

$$v_2 = \frac{1}{30}\left(\frac{r_1}{r_2}\right)^2 v_1$$

$$\Rightarrow v_2 = \frac{1}{30} \times \left(\frac{0.8 \times 10^{-2}\,\text{m}}{0.4 \times 10^{-2}\,\text{m}}\right)^2 \times (0.33\,\text{ms}^{-1})$$

$$v_2 = 0.044\ m\,s^{-1}$$

Numerical Problems

1. A geologist finds that a moon rock whose mass is 7.2 kg has an apparent mass 5.88 kg when submerged in water. Calculate the density of the rock
2. The density of ice is 971 Kgm^{-3} , and the approximate density of seawater in which an iceberg floats is 1025 Kgm^{-3}. what fraction of iceberg is under the water surface.
3. An iceberg having volume of 2060 cc floats in sea water of density 1030 Kgm^{-3} with a portion of 224cc above the surface, calculate the density of ice.
4. A 25 cm thick block of ice floating on fresh water can support an 80 kg man standing on it, what is the smallest area of an ice block?
5. A string supports a solid iron object of mass 180gm totally immersed in a liquid of density 800 Kgm^{-3} . The density of iron is 8000 Kgm^{-3}. calculate the tension in the string.
6. A piece of gold-aluminium alloy weights 100 gm in air and 80 gm in water. What is the weight of gold in the alloy if the relative density of gold is 19.3 and that of aluminium is 2.5.
7. An alloy of mass 588gm and volume 1000cc is made of iron of density 8.0 gm/cc and aluminium of density 2.7 gm/cc. calculate the proportion by i) volume ii) mass of the constituents of the alloy.

Numerical Problems

8. A rectangular plate of dimension 6cm $\times 4cm \times 2cm$ is placed vertical so that its largest side just touches the water surface. Calculate the downward force on the plate due to surface tension. (surface tension of water = $7.0 \times 10^{-2} Nm^{-1}$)
9. Calculate the work done in breaking a drop of water of 2mm diameter into millions droplets of same size. (surface tension of water = $72 \times 10^{-3} Nm^{-1}$)
10. Find the work done required to break down a water of radius 5×10^{-3}m into eight drops of water, assuming isothermal condition. (surface tension of water = $72 \times 10^{-3} Nm^{-1}$)
11. Calculate the change in surface energy of a soap bubble when its radius decreases from 5cm to 1cm. (surface tension of soap solution = $2.0 \times 10^{-2} Nm^{-1}$)
12. A capillary tube of 0.3m diameter is placed vertically inside a liquid of density 800 Kgm^{-3}, surface tension of liquid is $5.0 \times 10^{-4} Nm^{-1}$ and angle of contact is 30°. calculate the height to which liquid rises in the capillary tube.
13. Caster oil at 20°C has coefficient of viscosity $2.42 Nsm^{-2}$ and density 940 Kgm^{-3}. Calculate the terminal velocity of a steel ball of radius 2mm falling under gravity in the oil, taking the density of steel as 7800 Kgm^{-3}.
14. Calculate he magnitude and direction of the terminal velocity of an 1mm of radius air bubble using in an oil of viscosity 0.2 $Nm^{-2}S$? And specific gravity of 0.9 and density of air $1.29 kg/m^3$.

Continue

15. What is he terminal velocity of a steel ball falling through a tall jar containing glycerine? The densities of the steel ball and glycerine are 8.5 g/cc and 1.32 g/cc respectively and the viscosity oof the glycerine is 0.85 poise and the radius of the steel ball is 2mm.
16. Two drops of same liquid of same radius are falling through air with steady velocity of 2m/s. If the two drops coalesce what would be the terminal velocity?
17. Eight spherical raindrops of equal size are falling vertically through air with terminal velocity of 0.150 m/s. What would be the terminal velocity if these three drops were to coalesce to for a large spherical drop?
18. Glycerine flows steadily through a horizontal tube of length 1.5m and radius 1.0 cm. If the amount of glycerine collected per second at one end is $4.0 \times 10^{-3} kgs^{-1}$, what is the pressure difference between the two ends of the tube? (Density of glycerine = $1.3 \times 10^{3} kgm^{-3}$ and viscosity of glycerine = $0.83Nsm^{-2}$.)
19. Water flows steadily through a horizontal pipe of non-uniform cross-section. If the pressure of water is $4 \times 10^{4} Nm^{-2}$at a point where the velocity of flow is $2m/s$ and cross section is $0.02m^{2}$, what is the pressure at a point where cross section reduces to $0.01m^{2}$?
20. If the wind blows at 30m/s over the house, What is the net force on the roof if its area is $300m^{2}$? ($Density\ of\ air = 1.29kgm^{-3}$)

Thank you!