Fluid statics

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Hydrostatics: The study of fluid at rest is called hydrostatics.

Fluid: The substance that can flow from one point to another is called fluid. Both liquid & gases are taken as fluid.

Relative density (Specific gravity):
The absolute density of a substance with respect to that of water at 4°C is called Relative density of substance & given as

$$\rho_r = \frac{\rho}{\rho_w \text{ at } 4°C}$$

, ρ = absolute density
ρ_w = density of water.

Pressure: Normal force acting per unit area is called pressure. Alternatively, thrust per unit surface area is called pressure.

$$\therefore \text{Pressure} = \frac{\text{Normal force}}{\text{Area}} = \frac{\text{thrust}}{\text{Area}}$$

Unit is pascal and dimension is $[M^1 L^{-1} T^{-2}]$

Liquid pressure: The pressure exerted by the liquid on the wall of the container is called liquid pressure. It depends upon
i) density of liquid (d)
ii) height of liquid (h)
iii) Accelaration due to gravity (g).
But, liquid pressure doesn't depend

on the shape of liquid vessel and area of its base.

Mathematically,

$$P = hdg$$

Properties of liquid pressure:

i) Pressure at a point inside the liquid are equal in all direction.

ii) Pressure at a point inside liquid is independent on the shape of container. It is equal at equal height from free surface.

iii) Liquid pressure is greater at bottom than other points. So lower part of water reservoir is made thick.

Thrust: Force acting on a surface normally is called thrust.

Upthrust: When a body is partially or completely immersed in the liquid, the liquid exerts an upward force on it is called upthrust.

∴ Upthrust = Weight of body in air − Weight of body in water.

$$\text{Upthrust} = W_a - W_w$$

Also,

Upthrust = Weight of liquid displaced.

Archimedes' Principle: When a body is fully or partially immersed into fluid, it

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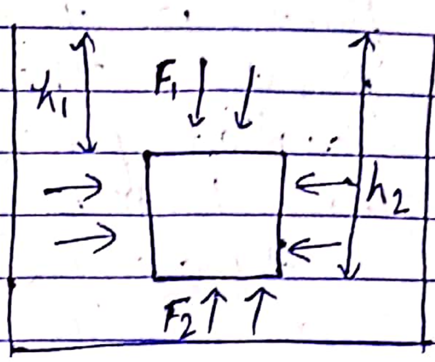
experiences upthrust due to fluid which is equal to the weight of fluid displaced.

Let us consider a cubical block (cuboid) of face area 'A' is fully immersed into the liquid of density (ρ) as in figure.



Here force acting on the sides of cuboid cancel each other and difference in force acting on its lower surface & upper surface gives upthrust.

Now,

downward force on face $F_1 = P_1 \times A$
$= h_1 \rho g A$

~~Upthrust~~ Upward force $F_2 = P_2 \times A$
$= h_2 \rho g A$

$\therefore$ Resultant upward force $= F_2 - F_1$
$= h_1 \rho g A - h_2 \rho g A$
$= \rho g A \ (h_2 - h_1)$
$= \rho g V$

Since, $A(h_2 - h_1) = V$ = volume of cuboid
= volume of liquid displaced

$\therefore$ Upthrust $= \rho g V$

Upthrust $= mg$

Upthrust = weight of liquid displaced

Note: ρV = mass of liquid displaced

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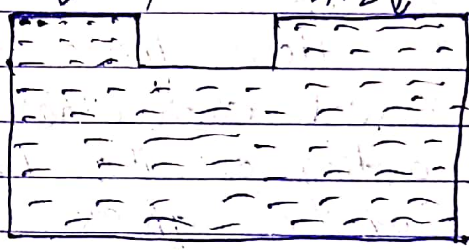
Pascal's law: It states that pressure applied at any point of a container enclosing fluid is transmitted equally in all it's point. This law is used to magnify applied force.

Eg: Hydraulic press, hydraulic break, Hydraulic Jack etc are based in this law.



Here
Pressure at 1st piston = pressure at 2nd piston

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$F_2 = \frac{A_2 \times F_1}{A_1}$$

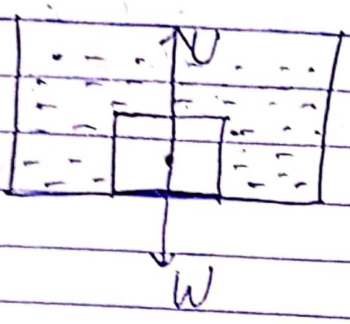
This shows that Force F_1 (applied force) is magnified because A_2 is greater than A_1.

Law of floatation: It states that, " A body in a liquid displaces the liquid equal to the weight of the body."

If a body is immersed in a liquid completely, it's weight 'W' acts vertically downward and Upthrust (U) due to displaced liquid acts vertically upward.

There are 3 possible cases:-

i)



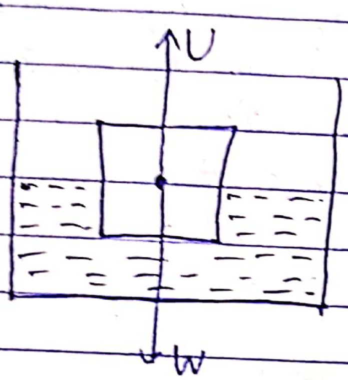
If ~~the~~ $W > U$

it sinks.

$$\rho V g > \rho_l V g$$
$$\rho > \rho_l$$

that means objects sinks if its density is greater than density of liquid.

ii)



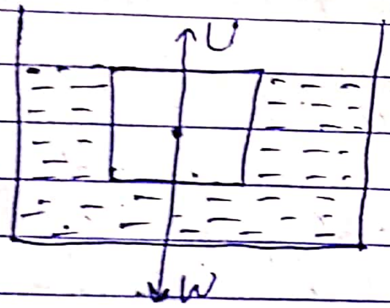
If $W < U$

it will float

Also, $\rho V g < \rho_l V g$

$$\rho < \rho_l$$

iii)



If $W = U$

it will just remain ~~float~~ sinks and remain inside liquid

$$\rho V g = \rho_l V g$$
$$\rho = \rho_l$$

Equilibrium of floating Bodies

i) Centre of Buoyancy: It is the point at which the centre of gravity of the displaced liquid lie. The buoyant force or upthrust acts vertically upward from the centre of buoyancy.

ii) Meta Centre: Meta centre of floating body on the liquid is the point of intersection of the

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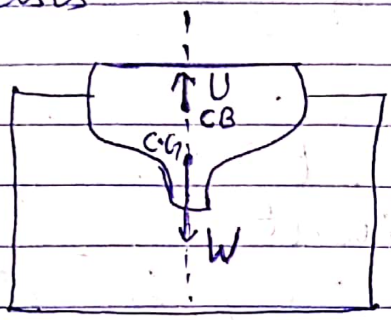
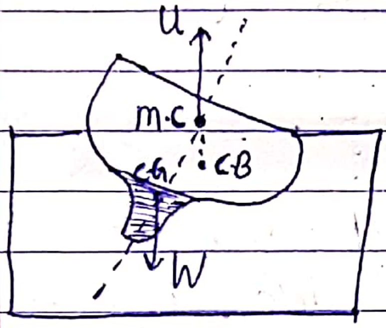
vertical line passing through CB and original line. Meta Centre lies only for floating bodies and helps to determine stability of floating bodies.

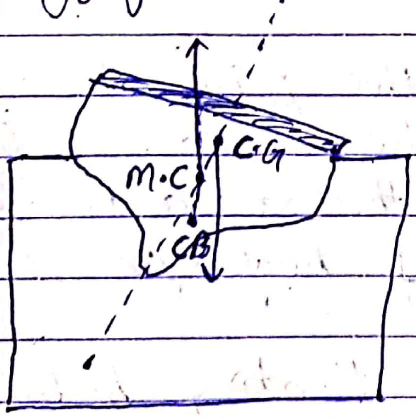
Cases

i) 

If C.G of body and C.B of displaced liquid both lie on the same vertical line then the floating body is stable.

ii) 

If C.G of body lies below C.B of liquid then M.C of body lie above C.G of floating body due to which W and U acting at C.G and C.B forms couple & brings the floating body back to equilibrium. That is why ships are made heavy and luggage and cargo are kept at bottom.

iii) 

If C.G of body lies above C.B of liquid then M.C lies below C.G due to which W and U acting at C.G and C.B rotates more and takes it away from equilibrium position making it unstable.

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that is why people are not allowed to stand on boat.

Surface tension: The property of liquid at rest by virtue of which the surface behaves like a stretched membrane & tries to occupy minimum surface area is called surface tension.

Mathematically, It is the force per unit length of an imaginary line which is drawn on the surface of liquid.

$$T = F/l$$

Its unit is N/m and dyne/cm.
Dimension is $[MT^{-2}]$

Here
$F = T \times 2l$

Since force is acting on both sides of thread so we take lenght '2l'.

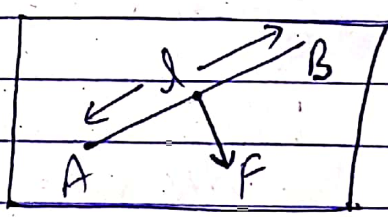

Fig: Surface tension on free surface of liquid

Intermolecular force:

A body is made up of large number of small particles called molecules. Each molecules attract other neighbouring molecules with a certain force called intermolecular force. It is of 2 types—

i) Cohesive force: The force of attraction between molecules of same substance (liquid)

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is called cohesive force. This force is maximum in solid, medium in liquid & least in gas.

ii) Adhesive force: The force of attraction between molecules of different substance is called adhesive force. It is different for different material.

Adhesive force of gum with paper is greater than that of water with paper.

Some examples

i) Water wets glass because adhesive force betn molecules of water & glass is greater than cohesive force between water molecules.

ii) When we write on board by chalk, it is written on board because adhesive force betn chalk and board molecules is greater than cohesive force betn ~~than~~ chalk molecules.

Molecular theory of Surface tension:

The molecules of a liquid attract each other with a force of cohesion (cohesive force). Now, Consider a molecule 'A' lying well below the liquid surface. It is attracted by all neighbouring molecules giving resultant force on molecule 'A' is zero.

Similarly, Consider another molecule 'B' on liquid surface. It is attracted by

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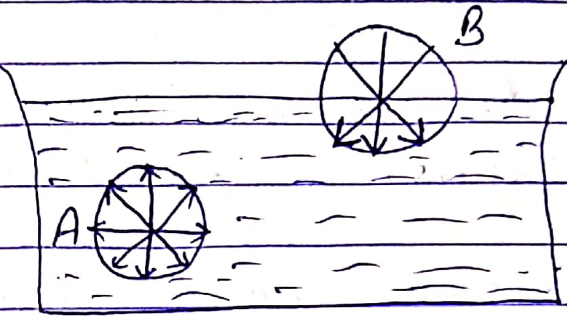
cohesive force by molecules below the liquid Surface, resulting downward force. So, molecules on the liquid Surface experiences maximum downward force. If we wish to bring a molecules from interior part to the surface, we have to do work, which is stored in the form of potential energy. In equilibrium state, liquid has minimum potential energy. It is possible only when there is minimum number of molecules on the Surface. Therefore, liquid surface tends to contract like an elastic membrane due to Surface tension.



Application of Surface tension

i) Use of detergent powder for washing the clothes.

ii) Oil rises up in oil lamp to tip of wick end.

iii) Low surface tension oil used for lubrication of machine part to reduce the friction.

iv) Use of towel to dry our body. & hair.

v) Mosquitos breed on the free Surface of stagnant water ~~containing~~ ~~oil~~. Because oil has less surface tension, when oil is spread in water, it cannot lay eggs.

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Note: Molecular Range: Maximum distance ~~between~~ upto which one molecule can attract another molecule

Sphere of influence: is the sphere whose radius is equal to molecular range.

Surface film: layer of the liquid whose thickness is equal to molecular range.

Surface Energy: The free surface of liquid has the tendency to contract to occupy minimum surface area. To increase the surface area, we have to do some work against the surface tension. This amount of workdone is stored in the form of surface energy in the liquid.

So, the surface energy is defined as the amount of workdone per unit increase in surface area of surface film.

ie. Surface Energy = $\frac{\text{workdone in increasing the surface area}}{\text{increase in surface area}}$

$$\sigma \text{ or } E = \frac{\Delta W}{\Delta A}$$

Relation between Surface Energy and Surface tension:

Consider, a rectangular frame of wire PBCQ ~~ABCD~~ in which the wire PQ is kept movable as in fig. let us dip this frame in a soap solution

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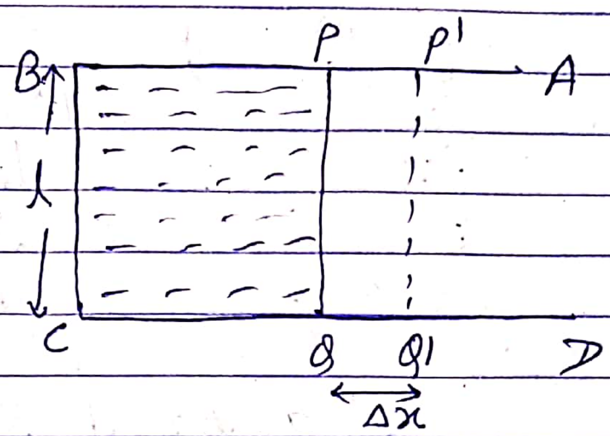
then thin Soap film is formed which pulls the wire PQ inward.
Let T be the Surface tension of the film and 'l' be length of wire BC. and the force due to ~~the~~ Surface tension is



$$F = 2l \times T$$
$$F = T \times 2l \quad ---(i)$$

Suppose the wire is transferred to P'Q' by covering the distance of Δx. Then increase in surface area is given by

$$\Delta A = 2(l \times \Delta x)$$

then

Amount of workdone to increase surface area is

$$\Delta W = F.\Delta x$$
$$\Delta W = T \times 2l \times \Delta x$$

From defn

$$\sigma = \frac{\Delta W}{\Delta A} = \frac{T \times \cancel{2l} \times \cancel{\Delta x}}{\cancel{2 \times l} \times \cancel{\Delta x}}$$

$$\boxed{\sigma = T}$$

that shows surface tension and surface energy are numerically equal.

Excess Pressure on Curved Surface of a liquid:

We know, there is force of Cohesion and force of surface tension exerting tangentially

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to the liquid surface at rest.

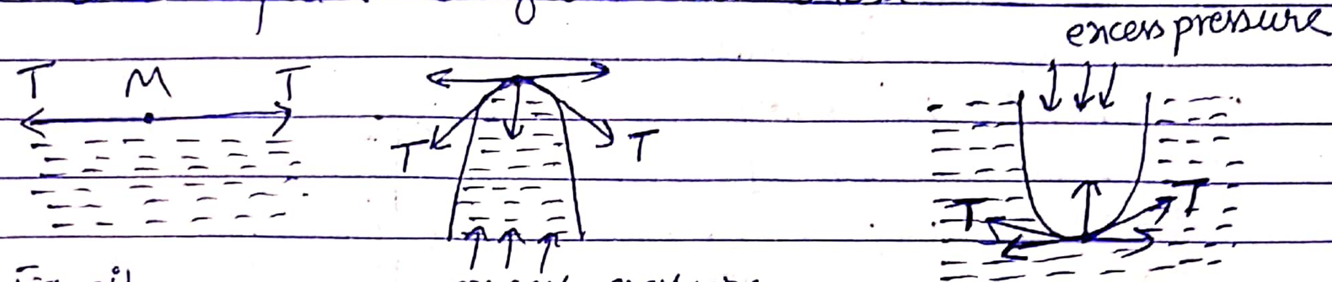


Fig (i) Fig (ii) Fig (iii)

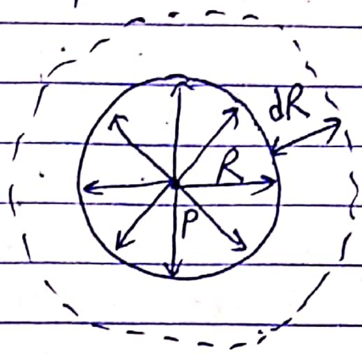
If a liquid surface is ~~plane~~ plane, a molecule M on the surface is acted by surface tension 'T' tangentially in all direction, resulting force on it is zero which is shown in fig (i).

Similarly, if we take a curve liquid surface, the molecule M is acted upon by surface tension (T, T) such that it's horizontal component on it will be zero and vertical components (downward) add up which is shown in fig (ii).

To be liquid in equilibrium position, excess force is applied to balance resulting downward force which results in excess pressure.

Similarly, in concave upward liquid surface, there is a resultant upward force acting on the molecule 'M' due to surface tension. To balance it there should be excess pressure. Hence for a curved surface of liquid in equilibrium, the pressure on its convex side is less than the pressure on its concave side.

Excess pressure inside a liquid Drop :



Consider a liquid drop of radius R lie on the surface of liquid. Due to surface tension, there is resultant force acting inward perpendicular to the surface. As a result this pressure inside the drop will be greater than the pressure outside. The excess pressure inside the drop will provide a force acting outward perpendicular to the surface which balances the surface tension force. Let 'T' be surface and 'p' be excess pressure inside the drop due to which the radius of drop increases to R+dR.

So, workdone = Force × displacement
= Excess pressure × Area × displacement
$= P \times 4\pi R^2 \times dR$
$= 4\pi P R^2 dR$ ---- (i)

Increase in surface area of drop $= 4\pi(R+dR)^2 - 4\pi R^2$
$= 4\pi(R^2 + 2RdR + dR^2) - 4\pi R^2$
$= \cancel{4\pi R^2} + 4\pi 2RdR + 4\pi dR^2 - \cancel{4\pi R^2}$
$= 8\pi R dR$

(since dR is very small, dR^2 can be neglected)

Increase in surface energy $= 8\pi R dR \times T$

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Surface energy (σ) = $\frac{\text{Workdone}}{\text{Increase in surface area}}$

$$T = \frac{4\pi PR^2 dR}{8\pi R dR}$$

$$T = \frac{PR}{2}$$

$$P = \frac{2T}{R}$$

or, $P_{in} - P_{out} = \frac{2T}{R}$ (excess pressure)

where P_{in} = Pressure inside the drop
P_{out} = Pressure outside the drop

⇒ Excess pressure of air bubble

Since the air bubble has only one free surface, so the excess pressure inside it is given by

$$P = P_{in} - P_{out} = \frac{2T}{R}$$

⇒ Excess pressure inside liquid bubble / Soap bubble

Since, the liquid bubble has two free surfaces. So the excess pressure inside it is given by –

$$P = P_{in} - P_{out} = \frac{4T}{R}$$

We know, excess pressure inside water drop is $P = \frac{2T}{R}$ and that of water bubble is $P = \frac{4T}{R}$

Due to excess/greater pressure inside bubble

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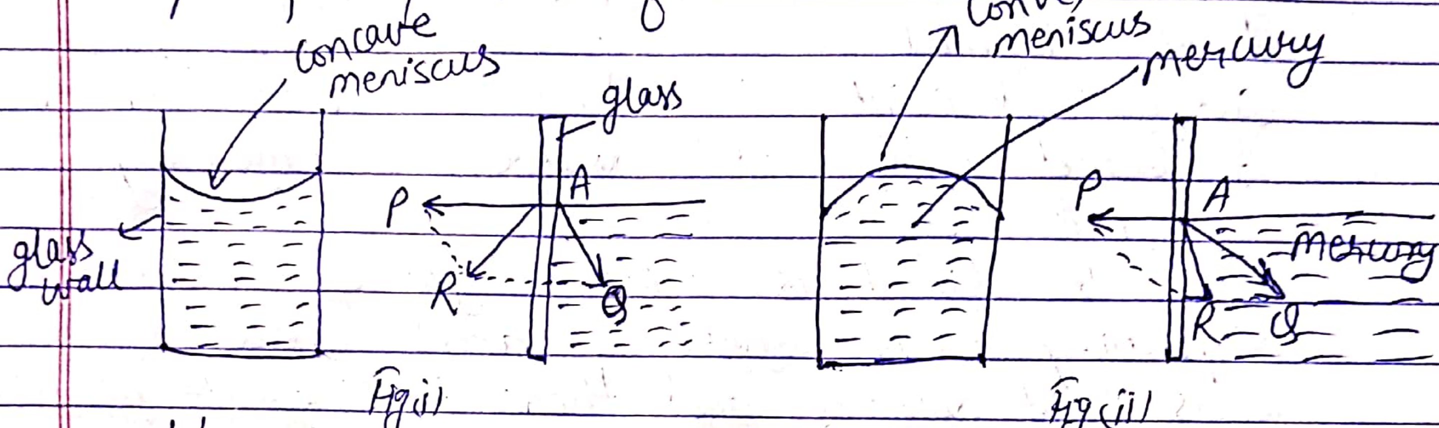
than in drop, bubble doesn't exist as same radius of drop.

Shape of liquid surface:



Usually liquid surface will be curved at the point of contact with a solid. The curved surface of liquid is called meniscus. Water forms concave meniscus whereas mercury forms convex meniscus which are shown in figure.

Fig (i) shows water in contact with a glass plate (wall). In the molecule A of water at point of contact, adhesive force is greater than cohesive force such ~~as~~ that resultant force are directed outward. As a result water ~~gets~~ wets the glass wall.

In fig (ii) shows mercury in contact with a glass wall. There is large cohesive force than adhesive force on the molecule A at the point of contact resulting inward resultant force 'R'. As a result mercury doesn't wett the glass wall.

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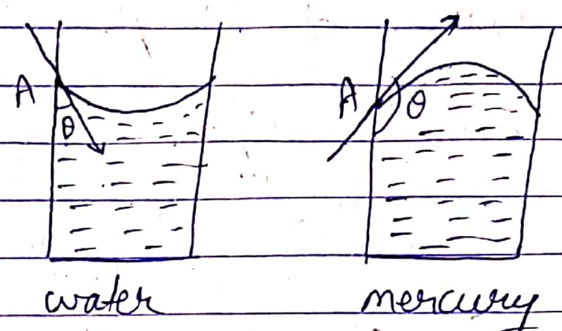
Angle of Contact :

The angle made by a tangent to the liquid surface at point of contact with the solid surface inside the liquid is called angle of contact / Capillary angle. There is acute angle which wets solid surface and there is ~~acute~~ obtuse angle which donot wets solid surface.



Water forms acute angle whereas mercury forms obtuse angle with the wall of container inside them.

Angle of contact for pure water is zero (it is ideal case). Generally angle of contact for water is about 8° and for mercury is 140° for glass surface.

Factors affecting angle of contact
i) Nature of solid and liquid in contact
ii) The medium that exists above the free surface of liquid.
iii) Temperature
iv) Impurities added to the liquid.

Capillarity : The rise or fall of a liquid in a tube of very fine bore is called capillary tube

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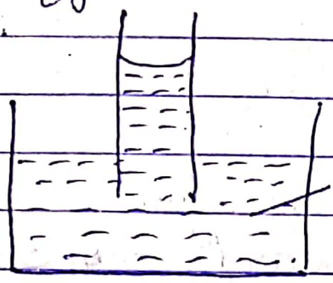
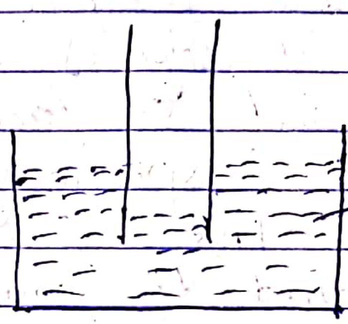
or capillarity action. When a tube of very fine bore having both ends open is dipped into water, the water rises above the water level in the tube. Similarly, when the tube is dipped into mercury, the mercury is depressed in the tube then its surface level is shown in fig ① & ②.





→ A blotting paper absorbs ink or water by Capillary action.

→ The tip of a nib of a pen is splitted to provide Capillary action for rise of ink.

→ A towel absorbs water by the capillary action method through the bores within fibres.

→ There is rise up oil through Capillaries of a Cotton wicks (thread).

Measurement of Surface tension by Capillary rise method

A tube having very fine bore is called capillary tube.

Let us Consider a Capillary tube of radius 'r' opens in both ends and dipped into water (liquid) which has a Concave meniscus as in fig.

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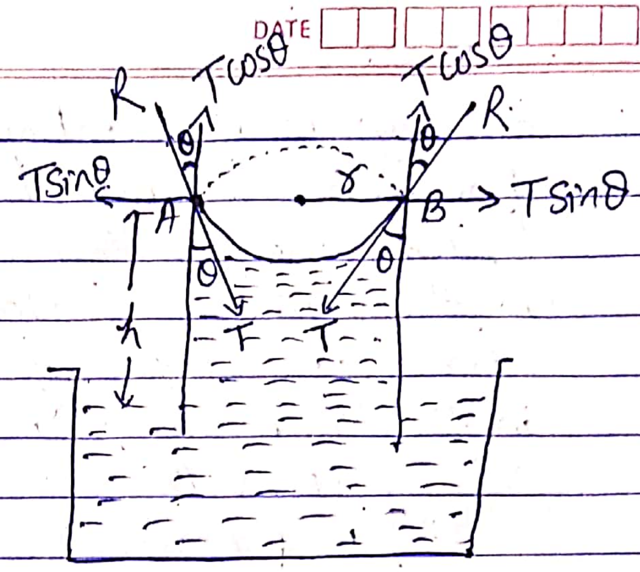

Consider a capillary tube of radius 'r'.

let θ, h, S and T be angle of contact, height of liquid column rise, density of liquid and surface tension respectively.

The surface tension force causes the liquid to exerts downward directed force 'T' on the wall of the tube. This force acts along the tangent at the point of contact A. From Newton's third law of motion the tube exerts an equal and opposite reaction R. The force (R=T) can be resolved into two components so that T sin θ and T cos θ are acted along horizontal & vertical resp. The horizontal component cancel each other whereas the vertical components are added which pulls the liquid upward. The force T cos θ acts along the whole circumference of the meniscus.

Now,

total upward force = $T \cdot \cos\theta \times 2\pi r$

Volume of liquid in the tube above the free surface of liquid = vol^n of cylinder of height 'h' and radius 'r' + vol^n of cylinder of height 'r' & radius 'r' − vol^n of hemisphere

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$$= \pi r^2 h + \pi r^2 r - \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$$

$$= \pi r^2 h + \pi r^3 - \frac{2}{3}\pi r^3$$

$$= \pi r^2 h + \frac{\pi r^3}{3}$$

$$= \pi r^2 \left(h + r/_3\right)$$

$\therefore$ Weight of liquid $= m \times g$

$= V \rho g$

$= \pi r^2 \left(h + r/_3\right) \rho g$

For Equilibrium

upward force = downward force

$$T \cos\theta \times 2\cancel{\pi}\cancel{r} = \cancel{\pi} r^{\cancel{2}} \left(h + r/_3\right) \rho g$$

$$T = \frac{r\left(h + r/_3\right)\rho g}{2\cos\theta}$$

If tube is very fine bore $r/_3$ can be neglected.

$$T = \frac{\rho g h r}{2\cos\theta}$$

$$\therefore h = \frac{2T\cos\theta}{\rho g r}$$

this is called ascent formula. In this case the angle of contact is acute.

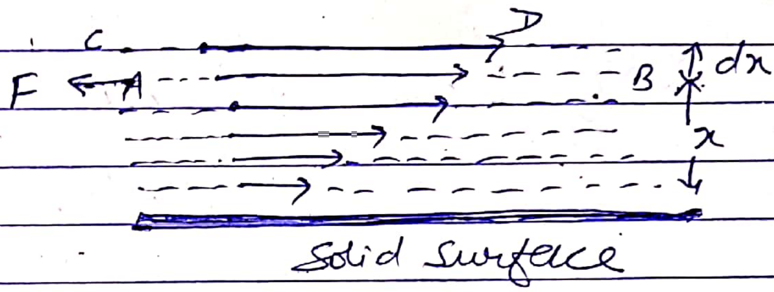
For pure water, $\theta = 0$

$$\therefore h = \frac{2T}{\rho g r}$$

Newton's law of Viscosity :

The property of fluid (liquid or gas) by virtue of which an internal force comes in play when the fluid is in motion & opposes the relative motion of its different layers is called viscosity. Eg- Honey, engine oil, lubricant, glue etc.

Let us consider a liquid flowing steadily on a fixed horizontal surface, the liquid layer which is in



contact with the surface is at rest. The velocity of upper layer goes on increasing and uppermost layer has maximum velocity. Now consider two liquid layers AB and CD at a perpendicular distance 'x' and '$x+dx$' from fixed surface. Let v be the velocity of layer AB and $v+dv$ be velocity of layer CD.

There is a viscous force 'F' between these two layer in backward direction. According to newton law viscous force F is

i) directly proportional to the area 'A' of contact of two liquid layers.

ie. $F \propto A$.

ii) directly proportional to the velocity gradient betn two layer of liquid. ie. $F \propto \frac{dv}{dx}$

Combining both,

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$$F \propto A\frac{dv}{dx}$$

$$F = -\eta A\frac{dv}{dx}$$

where η is proportionality constant called coefficient of viscosity. It depends upon nature of liquid. -ve sign shows that viscous force acts in opposite to motion of layer. This eq^n is called Newton's formula for viscosity.

we have

$$F = \eta A\frac{dv}{dx} \quad \text{(magnitude only)}$$

$$\eta = \frac{F}{A\frac{dv}{dx}}$$

SI unit of η is $N m^{-2} sec$ (or $kg m^{-1} s^{-1}$)

Dimension is

$$\eta = \frac{[MLT^{-2}]}{\frac{[L^2][LT^{-1}]}{[L]}}$$

$$\eta = [ML^{-1}T^{-1}]$$

Unit in cgs

$$\eta = \frac{1 dyne}{1 cm^2 s^{-1}} = dyne\, cm^{-2} sec \quad \text{(poise)}$$

ie. 1 poise = 1 dyne cm^{-2} sec

10 poise = 1 N sec m^{-2}

1 decapoise = 1 N sec m^{-2}

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If $A = 1 m^2$, $\frac{dv}{dx} = 1$ then

$$F = \eta.$$

Coefficient of viscosity of a liquid is defined as the viscous force acting per unit area of the layer having unit velocity gradient perpendicular to the direction of flow.

Poiseville's Formula :

Scientist poiseuille's studied the streamline flow (steady) of a liquid in a capillary tube and he gave some conclusions known as Poiseuille's formula (law).

According to him, volume of liquid flowing per second through capillary tube is

i) Directly proportional to the difference in pressure between two ends of the tube $(V \propto P)$

ii) Directly proportional to 4th power of radius of the tube.

$$V \propto r^4.$$

iii) inversely proportional to coefficient of viscosity of the liquid

$$V \propto \frac{1}{\eta}$$

iv) ~~From~~ Inversely proportional to lenght of tube

$$V \propto 1/l$$

Combining all

$$V \propto \frac{P r^4}{\eta l}$$

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$$V = \frac{KPr^4}{\eta l}$$

where K is proportionality constant
Experimentally $K = \pi/8$

then, $$V = \frac{\pi P r^4}{8 \eta l}$$

this eq^n is Poiseulles formula.

Derivation of Poiseulles formula by dimensional Analysis.

We know that, Volume of liquid flowing per second through a tube depend upon –

i) Pressure gradient (P/l) along the lenght of tube.
ii) Radius of the tube (r)
iii) Coefficient of viscosity (η)

ie. $$V \propto (P/l)^a r^b \eta^c$$

where a, b, c are constant

$$V = K (P/l)^a r^b \eta^c$$

where K is dimensionless constant

dimension of $V = [L^3 T^{-1}]$

" " $P/l = \frac{[ML^{-1}T^{-2}]}{[L]} = [ML^{-2}T^{-2}]$

$r = [L]$

$\eta = [ML^{-1}T^{-1}]$

putting these values,

$$[L^3 T^{-1}] = [ML^{-2}T^{-2}]^a [L]^b [ML^{-1}T^{-1}]^c$$

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$$[L^3 T^{-1}] = [M^{a+c} L^{b-c-2a} T^{-(2a+c)}]$$

Comparing power

$a + c = 0$ ——— (I)

$b - c - 2a = 3$ ——— (II)

$2a + c = 1$ ——— (III)

Solving (I), (II), (III)

$$a + c = 0$$
$$2a + c = 1$$
$$-a = -1$$
$$a = 1$$

$a + c = 0$ | $b - (-1) - 2(1) = 3$

$\therefore c = -1$ | $b = 4$

we get

$$V = K \left(\frac{P}{l}\right)^1 r^4 \eta^{-1}$$

$$V = \frac{K P r^4}{\eta l}$$

$$V = \frac{\pi P r^4}{8 \eta l} \quad \therefore \left(\because K = \pi/8\right)$$

Stream line flow :

Stream line flow of a liquid is that flow in which every particle of a liquid follows exactly the same path of it's preceeding particle & has the same velocity in direction or magnitude as that of it's preceeding particle while crossing through that point. Two stream line never cross to each other in a stream line flow.

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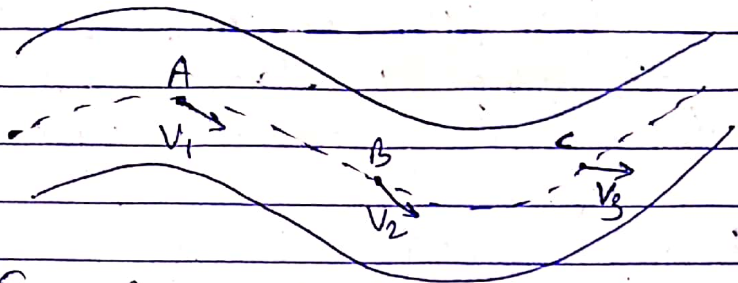

Fig: stream line flow:

Laminar flow: If a liquid is flowing over a horizontal surface in the form of layers having different velocities which donot mix with each other then such a flow is called laminar flow. The velocity of laminar flow is always less than critical velocity. In general, laminar flow is also called stream line flow.

Turbulent flow: When a liquid is flowing with a velocity greater than it's critical velocity, the motion of particle is called turbulent flow. The motion of the particles of liquid becomes irregular. In turbulent flow, the particle of liquid changes it's path and velocity continuously and haphazardly from point to point with variation of time.

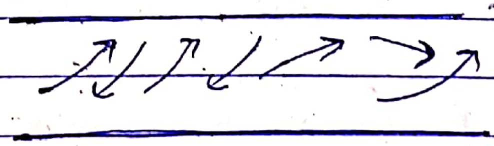

Fig: turbulent flow

Terminal velocity: The constant velocity of a body while moving through fluid is called terminal velocity.

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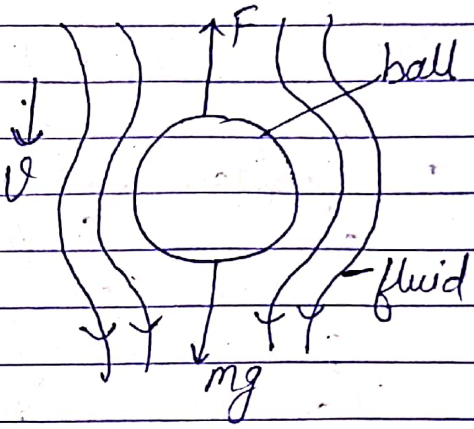
Critical velocity: The velocity of a liquid upto which the flow is laminar & above which it becomes turbulent is called critical velocity.

Stoke's law:
When a spherical metal ball is allowed to fall through a liquid, its velocity goes on increasing. There will be the point on the liquid beyond which velocity of the ball will be uniform. This uniform velocity acquired by the ball is called terminal velocity.



If η be coefficient of viscosity of liquid, r be radius of spherical ball and v be the terminal velocity of the ball. then,

$$\text{Viscous Force } (F) = 6\pi\eta r v$$

this is called stoke's law.

Derivation of stoke's law by dimensional analysis.

Viscous force 'F' depends upon

i) Coefficient of viscosity of the medium through which a spherical ball falls.

ie. $F \propto \eta^a$

ii) Radius of spherical ball

ie. $F \propto r^b$

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iii) terminal velocity acquired by the ball.
ie. $F \propto v^c$

Combining all

$$F \propto \eta^a r^b v^c$$

$$F = K \eta^a r^b v^c$$

where K is dimensionless Constant & a, b, c are constant.

Putting dimensions of F, η, r, v

$$[MLT^{-2}] = [ML^{-1}T^{-1}]^a [L]^b [LT^{-1}]^c$$

$$[MLT^{-2}] = [M^a L^{b+c-a} T^{-(a+c)}]$$

Comparing

$$a = 1$$

and $a + c = 2$

$$c = 1$$

and $b + c - a = 1$

$$b = 1$$

Now,

$$F = K \eta^1 r^1 v^1$$

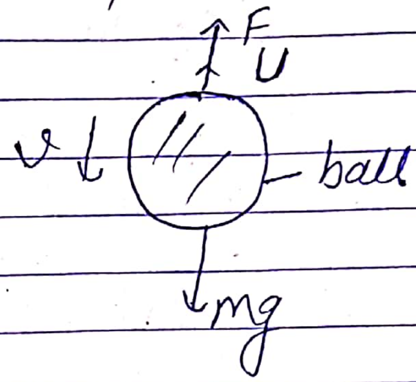
$$\therefore F = 6\pi \eta r v$$

Experimentally, stokes found that $K = 6\pi$.
this is Stoke's law.

Determination of coefficient of viscosity of liquid by a stoke's method.

When a spherical ball falls freely through a viscous medium, its velocity goes on increasing at first and finally a stage is reached at which its velocity will be uniform beyond it. In this stage, total resultant Force on the body will be zero. i.e. its weight is equal to the sum of viscous force and upthrust.

So,



$$F + U = W$$
$$F = W - U$$

we know,

$$F = 6\pi\eta r v$$
$$W = mg = \frac{4}{3}\pi r^3 \rho g$$
$$U = V\sigma g = \frac{4}{3}\pi r^3 \sigma g$$

where η = viscosity of liquid
r = radius of spherical ball
v = terminal velocity
ρ = density of ball
σ = density of liquid (fluid)

then

$$6\pi\eta r v = \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \sigma g$$
$$6\pi\eta r v = \frac{4}{3}\pi r^3 (\rho - \sigma) g$$
$$\eta = \frac{2r^2 g(\rho - \sigma)}{9v}$$

This is expression for coefficient of viscosity of fluid,

Equation of Continuity: Consider a tube AB of varying area of cross section through which an incompressible liquid is flowing steadily. Let

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v_1, ρ_1, a_1 and v_2, ρ_2, a_2 be the velocity of flow, density of liquid and area of cross section at point A and B resp.

Now,

the volume of liquid entering per second at A
$$= a_1 v_1$$
mass of liquid entering per second at A
$$= a_1 v_1 \rho_1$$
Similarly,

mass of liquid leaving per second at B
$$= a_2 v_2 \rho_2$$
if there is no loss of liquid,

mass of liquid entering = leaving
$$a_1 v_1 \rho_1 = a_2 v_2 \rho_2$$
For incompressible liquid $\rho_1 = \rho_2$
$$a_1 v_1 = a_2 v_2$$
$$av = \text{constant}$$
this is eqn of continuity

The eqn of Continuity States that, "If the area of cross section is large then the velocity of liquid becomes smaller and vice versa."

Energy of liquid

i) Kinetic Energy $= \frac{1}{2} mv^2$

K.E per unit mass $= \frac{1}{2} \frac{mv^2}{m} = \frac{1}{2} v^2$

ii) Potential Energy $= mgh$

P.E per unit mass $= \frac{mgh}{m} = gh$

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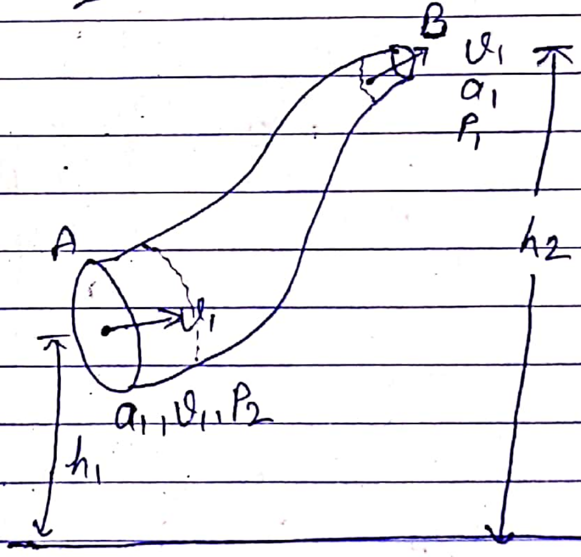
iii) Pressure energy per unit mass $= P/\rho$.

imp

Bernoulli's theorem:

It states that for the streamlined flow of an ideal liquid (incompressible & non viscous) the total energy (K.E + P.E + Pressure energy) per unit mass remains Constant at every cross-section through out the flow.

i.e. total Energy $E = P/\rho + \frac{1}{2}v^2 + gh$ = Constant



Consider a tube AB of varying area of cross-section through which on liquid liquid is in streamlined flow from A to B. Let P_1, a_1, v_1, h_1 & P_2, a_2, v_2, h_2 be pressure, area of cross section, velocity and height of point A and B respectively

Force acting on the liquid at end A $= P_1 a_1$
Displacement of liquid at end A in time $\Delta t = v_1 \Delta t$

$\therefore$ Workdone on liquid at end A

$W_1 = P_1 a_1 v_1 \Delta t$

Force given by liquid at end B $= P_2 a_2$

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displacement of liquid at end B in time Δt
$= v_2 \Delta t$

$\therefore$ Workdone by liquid at end B
$W_2 = P_2 a_2 v_2 \Delta t$

Net workdone on the liquid $= W_1 - W_2$
$W = P_1 a_1 v_1 \Delta t - P_2 a_2 v_2 \Delta t$
$W = P_1 V - P_2 V$
($\because$ From eqn of Continuity
$a_1 v_1 \Delta t = a_2 v_2 \Delta t$ = Volume of liquid)
$= V$

This workdone is converted into increase in potential and kinetic energy of the liquid.
Increase in P.E $= mgh_2 - mgh_1$
Increase in K.E $= \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$

where m is mass of liquid flowing per second.
So,

$$P_1 V - P_2 V = mgh_2 - mgh_1 + \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$

$$\frac{P_1 V}{m} - \frac{P_2 V}{m} = gh_2 - gh_1 + \frac{1}{2} v_2^2 - \frac{1}{2} v_1^2$$

$$P_1/\rho_1 - P_2/\rho_2 = gh_2 - gh_1 + \frac{1}{2} v_2^2 - \frac{1}{2} v_1^2$$

$$P_1/\rho_1 + gh_1 + \frac{1}{2} v_1^2 = P_2/\rho_2 + gh_2 + \frac{1}{2} v_2^2$$

i.e. $P/\rho + gh + \frac{1}{2} v^2$ = Constant

this is Bernoulli's theorem. In term of per unit volume can be written as

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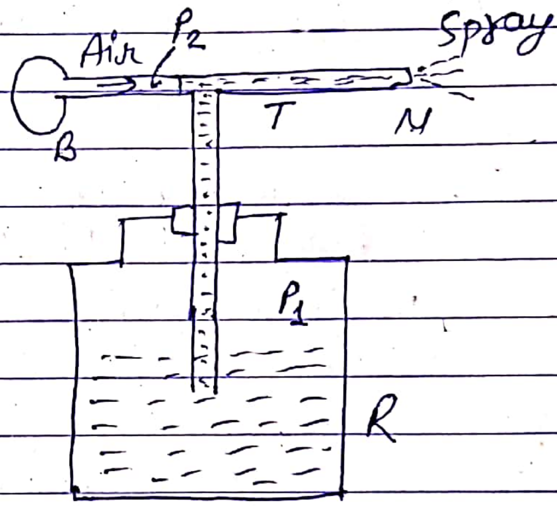
$$P + \rho g h + \frac{\rho v^2}{2} = \text{Constant}$$

When $h_1 = h_2$ i.e. for horizontal Surface

$$P + \frac{\rho v^2}{2} = \text{Constant}$$

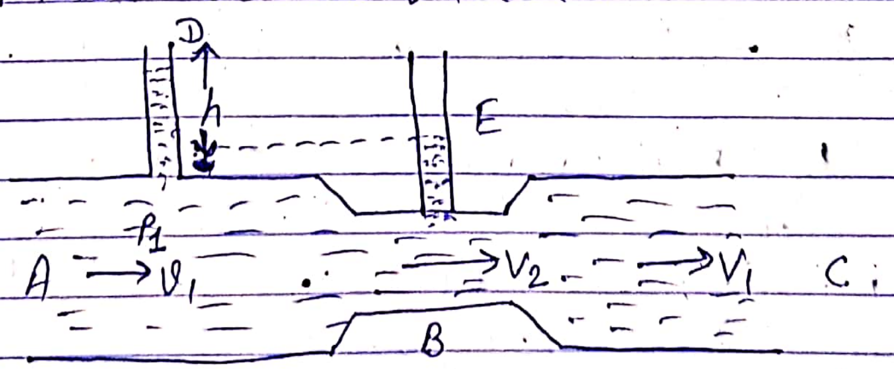
Application of Bernoulli's theorem:

i) Atomiser or Sprayer:



It Consists of a tube 'T' having rubber bulb 'B' at one end & tube is attached to a reserved wire. R As rubber bulb 'B' is pressed air in the tube flows out through the nozzle 'N' such that P_1 becomes greater than P_2. Due to this, liquid rises up in the tube 'T' and pushed out.

ii) Flow meter – Venturi Meter:



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It is a device used to measure the flow of liquid through pipe. It is based on Bernoulli's theorem. It consists of two pipe A and C of same cross section area connected by narrow tube B. There are two tubes D and E which measures difference in pressure of liquid flowing through A and B. Let P_1, a_1, v_1 and P_2, a_2, v_2 be pressure, area and velocity of flow at A & B.

$V = a_1 v_1 = a_2 v_2$ is constant

" volume of liquid flowing per second

Since,

$h_1 = h_2$

we have,

$$\frac{P_1}{\rho_1} + \frac{1}{2} v_1^2 = \frac{P_2}{\rho_2} + \frac{1}{2} v_2^2$$

$$\frac{(P_1 - P_2)}{\rho} = \frac{1}{2}(v_2^2 - v_1^2) \quad (\because \rho_1 = \rho_2 = \rho)$$

$$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2)$$

$$h\rho g = \frac{1}{2}\rho(v_2^2 - v_1^2)$$

$$2hg = \frac{V^2}{a_2^2} - \frac{V^2}{a_1^2}$$

$$2gh = V^2\left(\frac{a_1^2 - a_2^2}{a_1^2 a_2^2}\right)$$

$$V^2 = \frac{2gh\, a_1^2 a_2^2}{a_1^2 - a_2^2}$$

$$V = a_1 a_2 \sqrt{\frac{2gh}{a_1^2 - a_2^2}}$$

By knowing a_1, a_2, h we can know volume of liquid flowing per second.

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iii) Lifting of an aeroplane :

The shape of plane is slightly Convex upward & concave downward. Due to this air pressure above the plane will be low and below the plane will be high. Due to this difference in pressure the plane will be lifted up.