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Chapter- 18 Electromagnetic Induction

18.1 Faraday's laws; Induced electric fields

18.2 Lenz's law, Motional electromotive force

18.3 A.C. generators; Eddy currents

18.4 Self-inductance and mutual inductance

18.5 Energy stored in an inductor

18.6 Transformer.

Electromagnetic Induction:

- 18.1 State and show understanding of Faraday's law of electromagnetic induction
- 18.2 State and show understanding of Lenz's law
- 18.3 Discuss construction and working of A.C. generators
- 18.4 Define eddy currents, explain how they arise and give a few examples where eddy currents are useful and where they are nuisance
- 18.5 Describe self-inductance and mutual inductance and understand their uses
- 18.6 State the expression for energy stored in an inductor and use it wherever needed
- 18.7 Discuss the construction, working principle and importance of transformer
- 18.8 Discuss the sources of energy loss in practical transformer

Magnetic Flux

Magnetic flux at any surface is no. of magnetic lines of force passing normally through that surface.
Let B be the magnetic field and A be the area of the surface where the magnetic lines of force are normally incident. Then

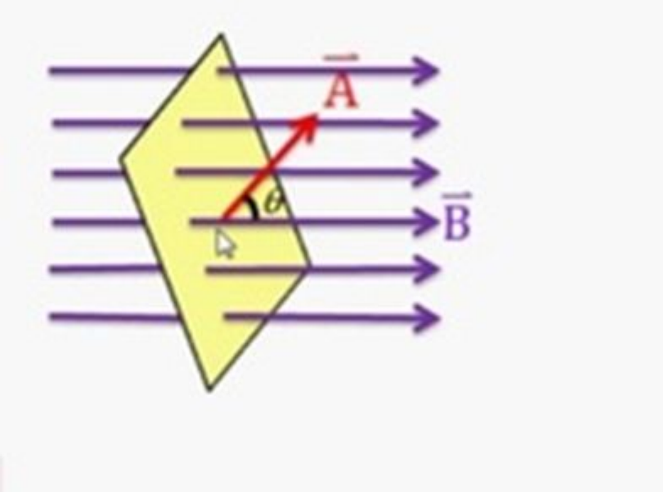
magnetic flux $(\Phi) = \vec{B} \cdot \vec{A}$

$= BA\cos\theta$

$\therefore \Phi = BA\cos\theta$

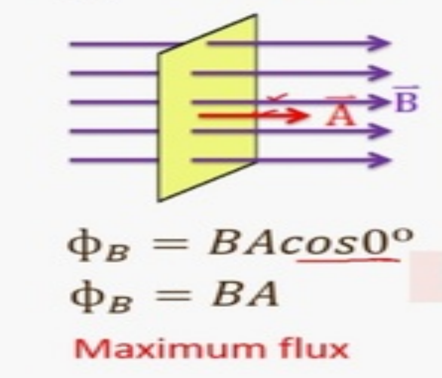
where, θ is the angle between the magnetic field lines and the normal (perpendicular) to A.

It is a scalar quantity. The S.I unit of magnetic flux is Weber(Tesla m^2).
CGS unit is Maxwell

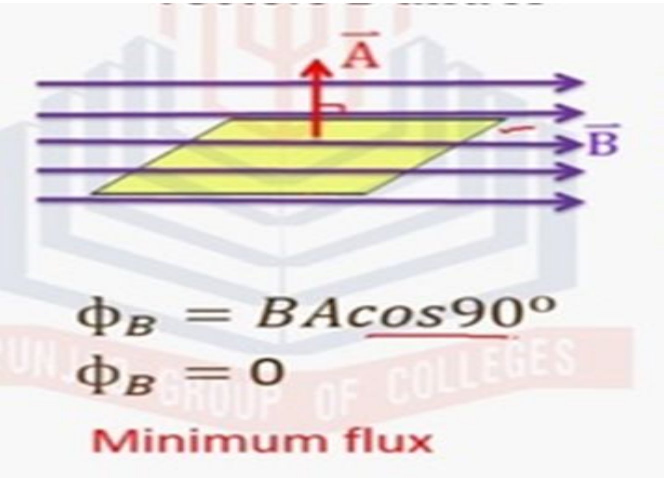


Special cases

i. When θ= 0°



ii. When $\theta = 90^\circ$



Magnetic Flux density or magnetic field strength or magnetic field

It is defined as the magnetic flux per unit area.

$$\text{i.e } B = \frac{\Phi}{A}$$

Its S.I unit is Tesla(Weber/m^2).

Electromagnetic Induction:

The phenomena of generation of an electric current due to change of magnetic lines of force (magnetic flux) is called electromagnetic induction. The produced current is called induced Current and the corresponding emf is called induced emf.

Faraday's Laws of electromagnetic induction

Faraday's laws of electromagnetic induction states that

i. Whenever the flux linking with the coil is Changed, an emf is induced. The induced emf lasts as long as the change in magnetic flux takes place.

ii. The magnitude of induced emf in a coil is directly proportional to the rate of change of magnetic flux.

i.e. $E \propto \frac{d\emptyset}{dt}$

Suppose the magnetic flux linking with the coil is changed from $\emptyset_1$ to $\emptyset_2$ and induced emf is E then

$$E \propto \frac{\emptyset_2 - \emptyset_1}{dt}$$

$$E = -k \frac{\emptyset_2 - \emptyset_1}{dt}$$

where, k is proportionality constant. In SI unit k=1. –Ve sign indicates that induced emf is opposite to change in flux.

$$E = -\frac{\phi_2 - \phi_1}{dt}$$

$$E = -\frac{d\phi}{dt}$$

for N no. of turns,

$$E = -N\frac{d\phi}{dt}$$

Note:

i. Induced current (I)= $\frac{1}{R}\frac{d\phi}{dt}$ **[E=IR]**

ii. Induced charge (q)= $\frac{d\phi}{R}$

1. A circular loop of area 0.015 m^2 is placed in a uniform magnetic field of 0.30T. If its plane makes an angle of 37^0 with field, then the magnetic flux through the loop is

a. Zero b. 2.7 mWb c. 3.6 mWb d. 45mWb

2. The magnetic field and number of turns of the coil of an electric generator is doubled then the magnetic flux of the coil will

a. Become half

b. Become two times

c. Become three times

d. Become four times

Question: The magnetic flux linked with a coil changes from 12 x 10^{-3} Wb to 6 x 10^{-3} Wb in 0.01 second. Calculate the induced emf.

Question: The magnetic flux linked with a coil changes from 12 x 10^{-3} Wb to 6 x 10^{-3} Wb in 0.01 second. Calculate the induced emf.

Answer: $\emptyset_1 = 12 \times 10^{-3}$ Wb , $\emptyset_2 = 6 \times 10^{-3}$ Wb

Induced emf (E) $= -\frac{d\emptyset}{dt} = -\frac{\emptyset_2 - \emptyset_1}{dt} = \frac{6 \times 10^{-3} - 12 \times 10^{-3}}{0.01} = \frac{6 \times 10^{-3}}{10^{-2}} = 0.6V$

3. In electromagnetic induction, the induced charge is independent of

(a) change of flux
(b) time
(c) resistance of the coil
(d) None of these

4. An induced e.m.f. is produced when a magnet is plunged into a coil. The strength of the induced e.m.f. is independent of

(a) the strength of the magnet
(b) number of turns of coil
(c) the resistivity of the wire of the coil
(d) speed with which the magnet is moved

5.Faraday's laws are consequence of the conservation of

(a) charge
(b) energy
(c) magnetic field
(d) both (b) and (c)

6. The magnetic flux through a coil is varying according to the relation,

$\phi = (4t^3 + 5t^2 + 8t^{-3} + 5)$ weber

Calculate the induced current through the coil at t = 2S if the resistance of the coil is 3.1 Ω.

Solution

We have,

$$\phi = 4t^3 + 5t^2 + 8t^{-3} + 5$$

$$\therefore \frac{d\phi}{dt} = 12t^2 + 10t - 24t^{-4}$$

$$= 12t^2 + 10t - \frac{24}{t^4}$$

$$\therefore \text{Induced e.m.f., } \varepsilon = \left|\frac{-d\phi}{dt}\right|_{t = 2\,S} \quad \text{[i.e. putting t = 2 S]}$$

$$= \left|12 \times 4 + 10 \times 2 - \frac{24}{16}\right|$$

$$= \left|48 + 20 - \frac{24}{16}\right| = 66.5\ \text{V}$$

Again, $|\varepsilon| = IR$

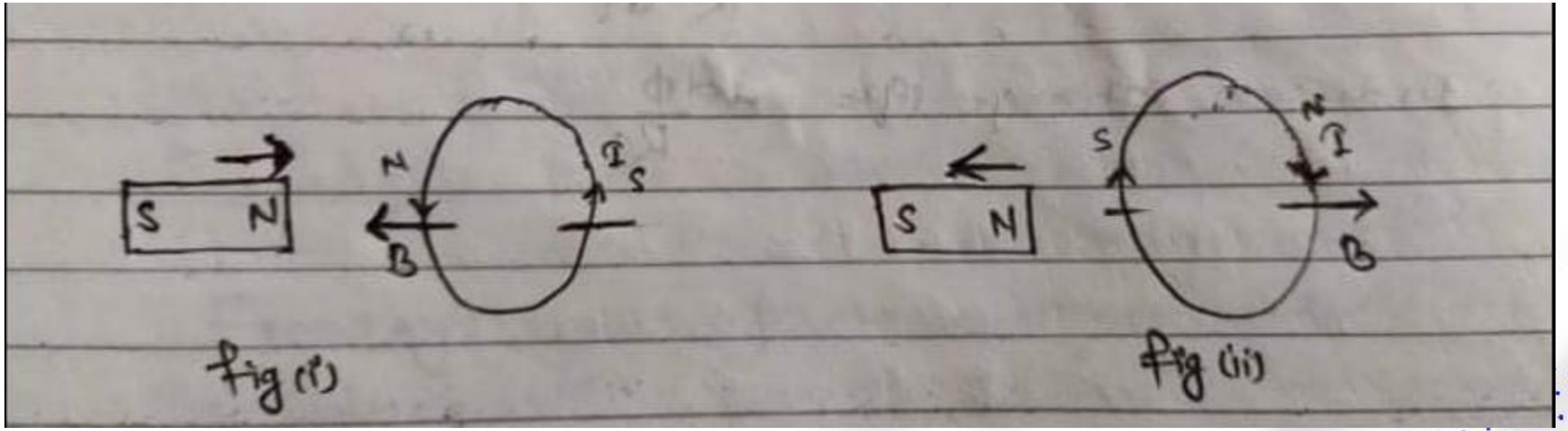
or $I = \frac{|\varepsilon|}{R} = \frac{66.5}{3.1} = 21.45$ A

∴ Induced current, I = 21.45 A

Lenz's Law

Lenz's law states that **"the direction of induced current is such that it opposes the causes which produces it"**. It gives the direction of flow of current due to induced emf.

Explanation:

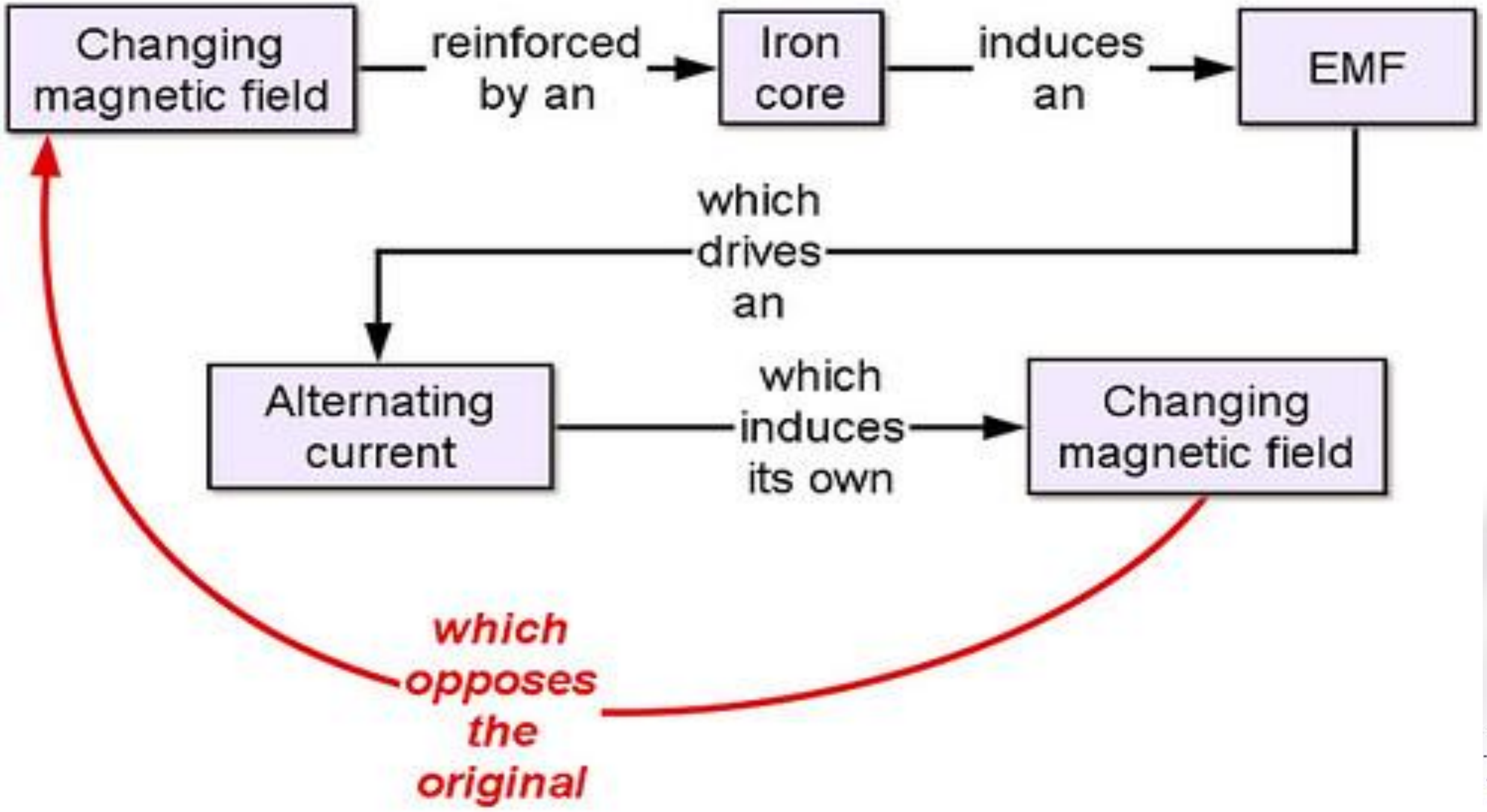


To explain the lenz's law, let us consider a bar magnet and circular coil where the north pole of bar magnet is pointing to the face of the coil. When the bar magnet is bought near to the coil, the flux linked with the coil is changed. As a result, an emf is induced and produces induced current. The current produce in the coil opposes the moving magnet as shown in figure (i).

However, if the bar magnet is taken away from the coil, the induced emf again produces the current but in opposite direction as before. So the face of the coil behaves as south pole tending to attract the magnet as shown in figure (ii).

Thus, in both cases the direction of induced current is such that it opposes the motion of the magnet.

Lenz' Law

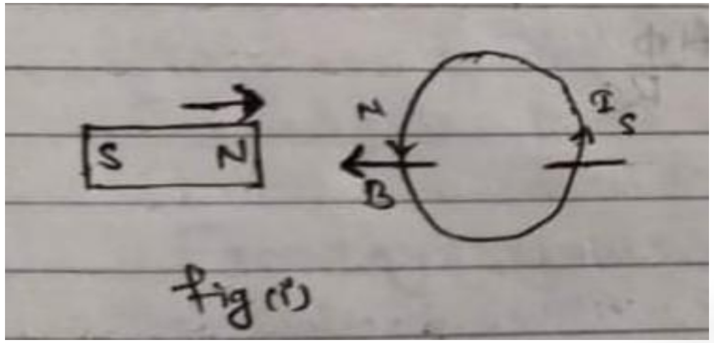

Changing magnetic field
reinforced by an
Iron core
induces an
EMF
which drives an
Alternating current
which induces its own
Changing magnetic field
which opposes the original

Q.N Show that Lenz law obeys the principle of conservation of energy.

OR

Lenz law follows the principle of conservation of energy. Explain.

Solution,



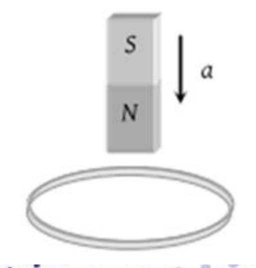
According to principle of conservation of energy "energy can neither be created nor be destroyed but it can only be changed from one form to another form"

When the bar magnet pointing north pole is moved towards the coil, flux linked with the coil is changed. As a result, an emf is induced and produces induced current. The induced current opposes(repels) the motion of magnet as shown in fig(i). Now, in order to move the bar magnet further towards the coil, an extra work has to be done against repulsive force. This work done is stored in the coil as the form of electrical energy. That means mechanical energy is converted into electrical energy (i.e. magnitude of induced emf). Hence, Lenz law obeys the principle of conservation of energy.

1. Lenz's law is an example of conservation of
a. Charge   b. Momentum   c. Mass   d. Energy

2. Lenz's law gives the direction of
a. Motion of conductor   b. Magnetic field   c. Induced current   d. Current in any electrical circuit

3. A bar magnet of length L is dropped inside a vertical copper pipe of length l (l <L); it will experience an acceleration 'a' such that;
a. $a>g$ b. $a=g$ c. $a<g$ d. $a=\frac{lg}{L}$



4. The current flows from A to B is as shown in the figure. The direction of the induced current in the loop is
(a) clockwise.
(b) anticlockwise.
(c) straight line.
(d) no induced e.m.f. produced.

Answer/Explanation

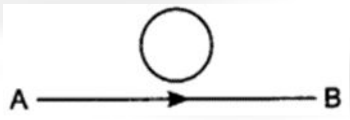
Answer: a

Explaination:

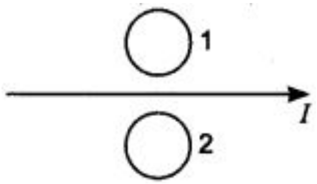
(a) By lenz's law, the induced current must produce inward flux to counter magnetic flux of AB. So induced current is clockwise in the loop.
Direction of magnetic field due to indicated current will be in upward direction passing through the coil. So, a current in coil will be induced so that it decreases the flux in the coil.
So, current in the clockwise direction will be induced.

5. In the given figure current from A to B in the straight wire is decreasing. The direction of induced current in the loop is

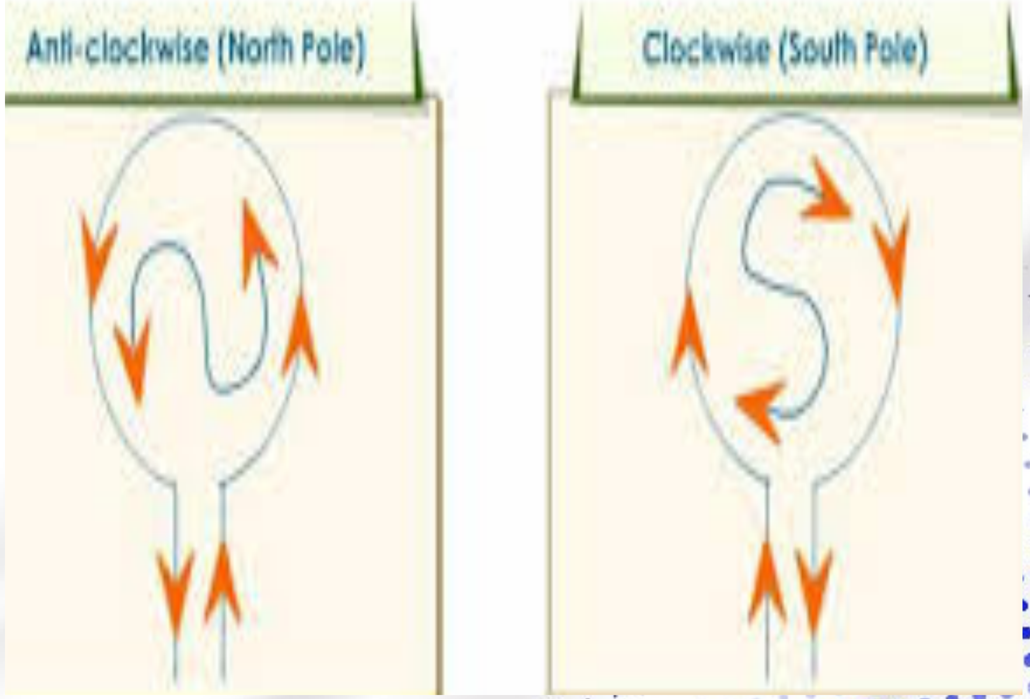
(a) clockwise
(b) anticlockwise
(c) changing
(d) nothing can be said



What is the direction of induced currents in metal rings 1 and 2 when current I in the wire is increasing steadily?



1. Clockwise
2. Anticlockwise



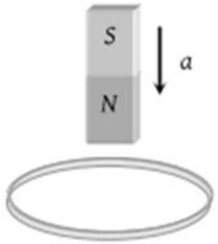
Homework

Short question

A copper ring is held horizontally and a bar magnet is dropped through the ring with its length along the axis of the ring. Will the acceleration of the falling magnet be equal to the acceleration due to gravity ? Explain

OR

A bar magnet falls through a metal ring. Will its acceleration be equal to g? Justify your answer.



Ans: when the magnet approaches the ring, the magnetic flux linked with the ring is changed as result an emf is induced in a ring and produces induced current in the ring. According to Lenz law, the current opposes the approach of magnet due to opposing nature of induced emf (induced current), hence the acceleration of the magnet becomes less than g.

Numericals

1. The magnetic flux passing perpendicular to the plane of coil is given by $\phi = 4t^2 + 5t + 2$, ϕ is in Weber and t is in seconds. Calculate the magnitude of instantaneous emf induced in the coil when t=3sec.

2. A coil of 100 turns each of area 2×10^{-3} m^2 has a resistance of 12 Ω. It lies in a horizontal plane in a vertical magnetic flux density of 3×10^{-3} Wbm^{-2}.What charge circulates through the coil of its end are short circulated and the coil is rotated through 180^0 about a diameter.

3. A coil of 100 turns and area $0.1m^2$ is in a magnetic field of induction 4×10^{-3} Wbm^{-2}. If the coil is reversed in a time of 50ms, calculate the induced emf in it.

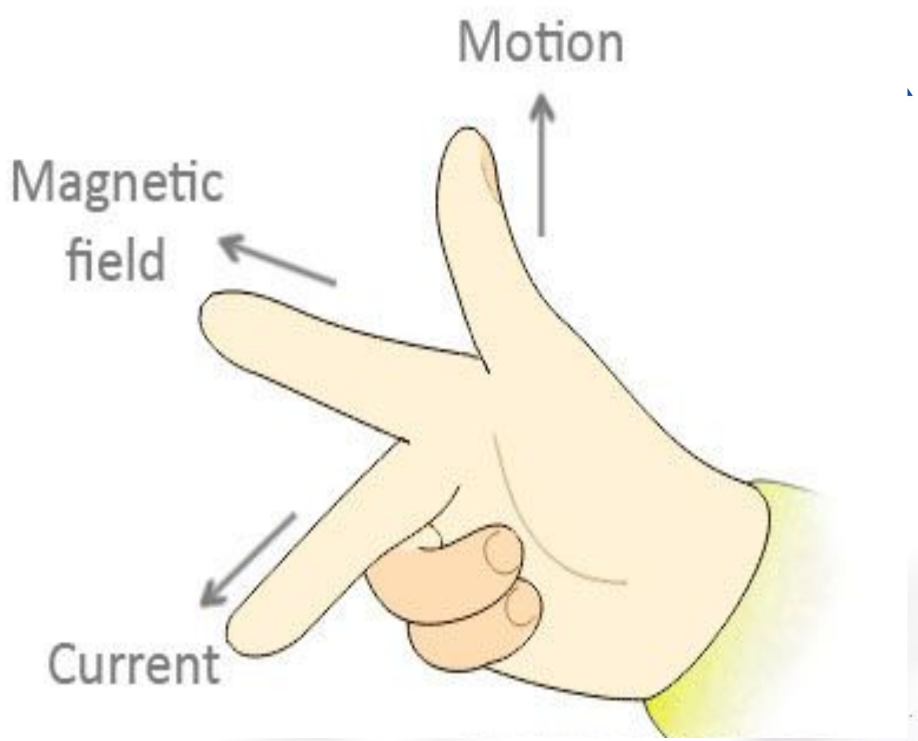
Fleming's Right Hand Rule

When thumb, fore finger and middle finger are mutually perpendicular to each other, then

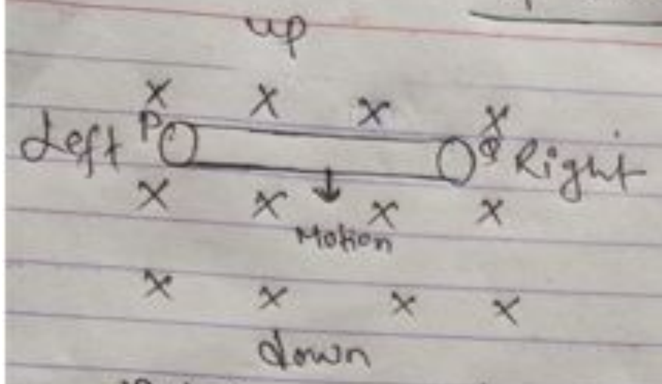
Thumb→ direction of motion of conductor

Fore finger → direction of magnetic field

Middle finger → direction of current



Use of Fleming's right hand rule to find direction of current

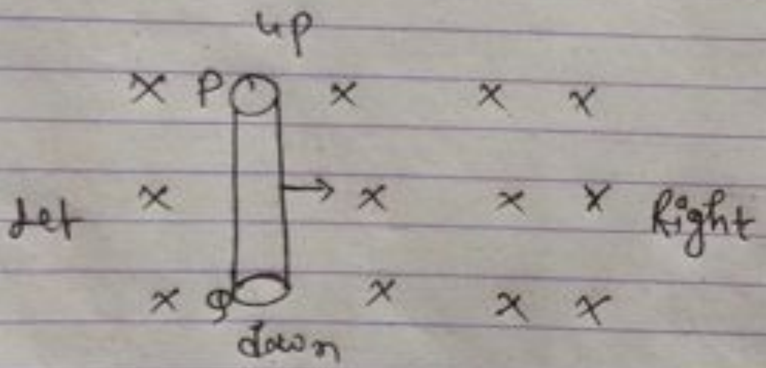


P→Q current (Left to right)

fig (i)

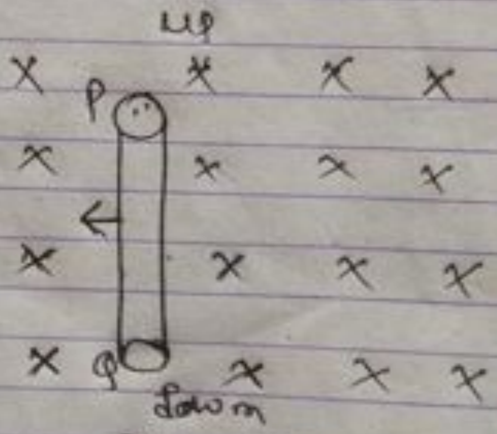
Q→P current (Right to left)

fig (ii)



Q→P (down to up)

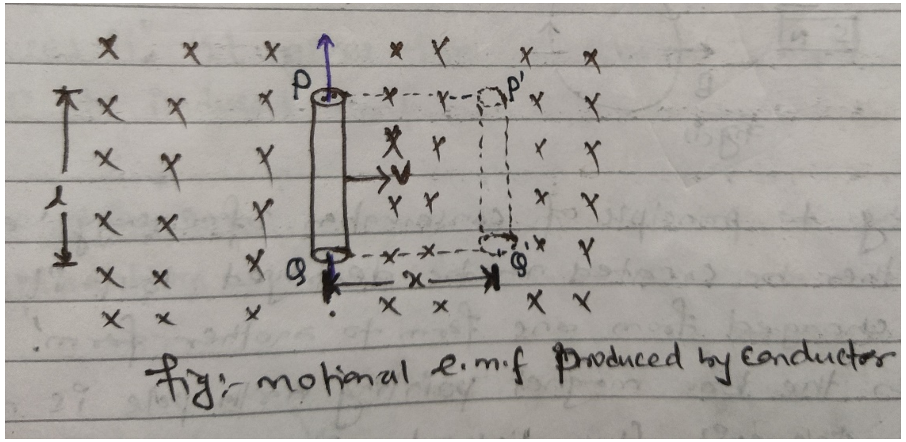
fig (iii)



P→Q (up to down)

fig (iv)

Induced emf in a straight conductor moving in uniform magnetic field(motional electromotive force)



- Let us consider a straight conductor of length *l* moving at right angle to the direction of uniform magnetic field B with velocity V as shown in above figure.

Let the conductor moves small distance x in time t. From the figure, the area swept out by the conductor is

$$A = lx \ldots\ldots\ldots\ldots\ldots\ldots\ldots(i)$$

Now, the flux cut out by the conductor in a magnetic field is

$$\Phi = B.A$$

$$= Blx \quad \text{(using (i))}$$

From Faraday's law of electromagnetic induction, the induced emf is equal to the rate of change of flux.

dØ

$$E = Bl\frac{d(x)}{dt}$$

$$\therefore \quad E = Blv$$

Hence, the induced emf in a conductor is

$$E = Blv$$

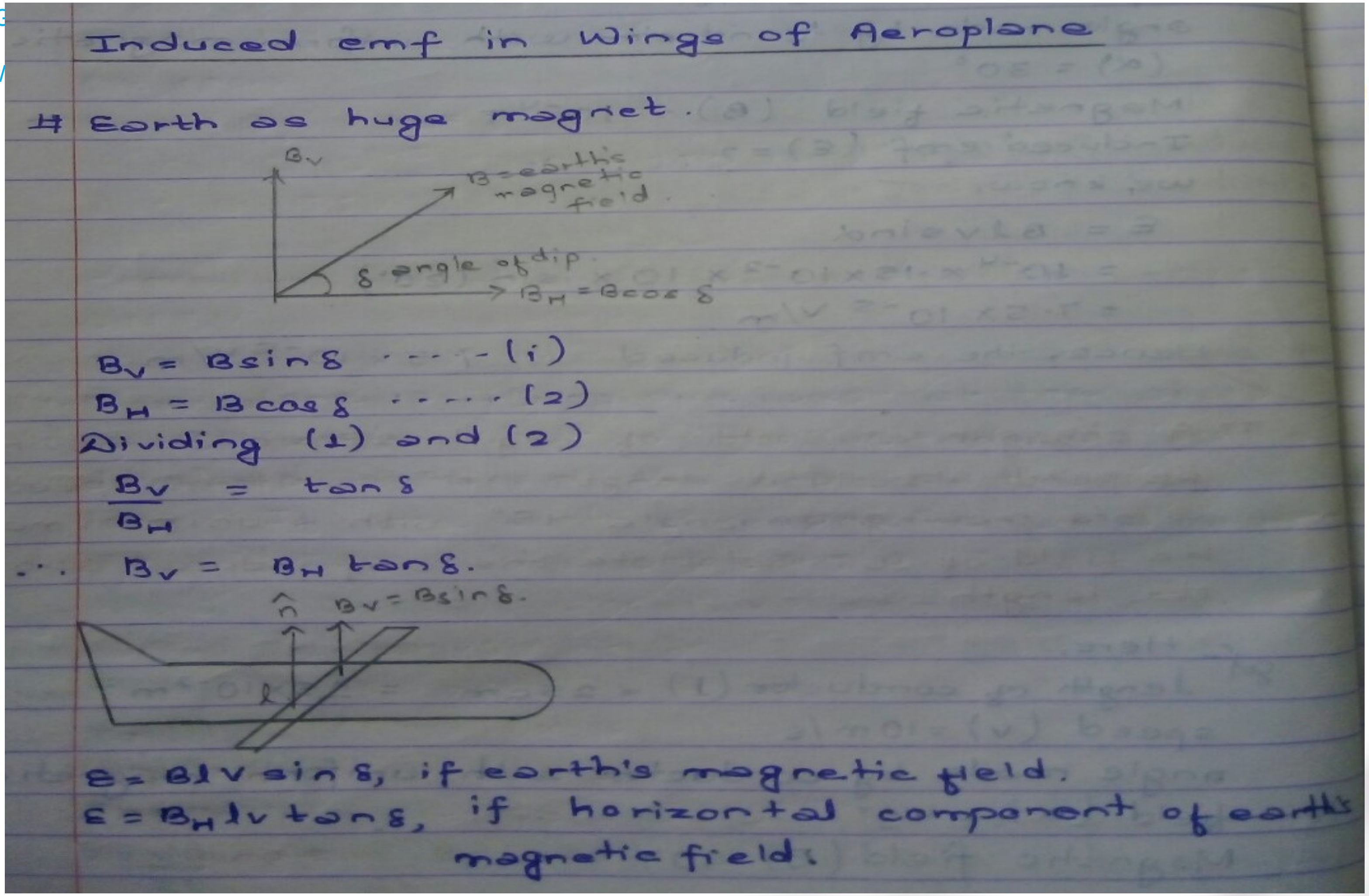
Special case :

If a conductor is placed making an angle θ with the direction of the field, then the induced emf is

$$E = Blv \sin\theta$$

Numerical

A straight conductor of length 15cm is moving with uniform speed of $10ms^{-1}$ making an angle of 30^0 with uniform magnetic field of 10^{-4} tesla. Calculate the emf induced across the length.



Induced emf in Wings of Aeroplane

\# Earth as huge magnet.

$B_V = B\sin\delta$ .... (i)

$B_H = B\cos\delta$ .... (2)

Dividing (1) and (2)

$$\frac{B_V}{B_H} = \tan\delta$$

$\therefore B_V = B_H \tan\delta.$

$\varepsilon = Blv\sin\delta$, if earth's magnetic field.

$\varepsilon = B_H l v \tan\delta$, if horizontal component of earth's magnetic field.

Note: Since only the vertical component of earth's magnetic field will cut the wings of plane perpendicularly, thus only this component will produce induced emf

- What is motional emf? On what factors does it depend?

Numericals

1. A jet plane is flying due west at the speed of 1800km/h. What is the voltage difference developed between the ends of wings 25m long. If the earth's magnetic field is 5x 10^{-4} T and dip angle is 45^0 .

2. A metal aircraft with a wing span of 40m flies with speed 1000km/h in a direction due east at constant altitude in a region of the northern hemisphere where the horizontal component of the earth's magnetic field is 1.6x 10^{-5} T and the angle of dip 41^0. Find the potential difference developed between the tips of the wing.
[0.155V]

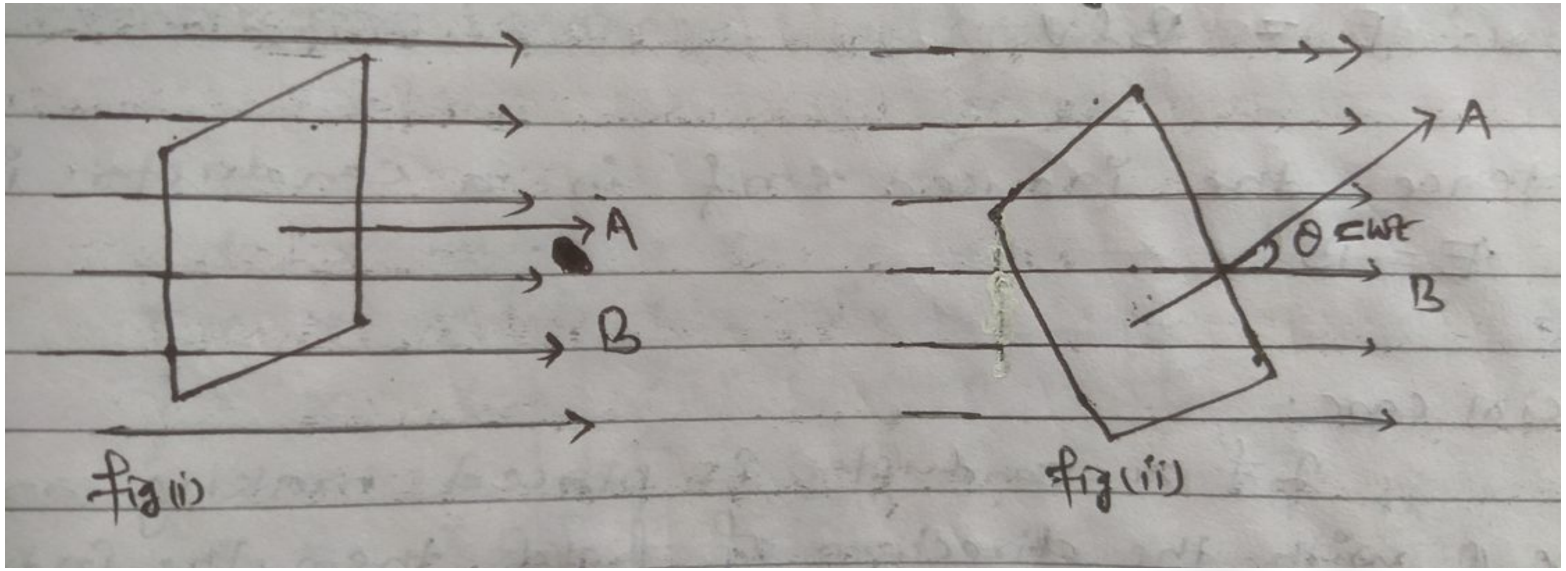
3. A jet airplane with a 75m wingspan is flying at 280 m/s. what emf is induced between wing tips if the vertical component of the earth's field is 3×10^{-5} T?

4. A straight wire 30m long moves at 2m/s perpendicularly through a 1T magnetic field.

i. Calculate the induced emf in a wire.

ii. The total resistance of the circuit of which the wire is part is 15Ω. What is the current?

Induced emf in a coil rotating in a uniform magnetic field



- Let us consider a rectangular coil having N number of turns and each having area A is placed in a uniform magnetic field B. At t=0, the axis of coil makes an angle 0^0 with the direction of magnetic field as shown in fig (i). At any instant, the coil is rotated anticlockwise direction by an angle θ with the direction of magnetic field as shown in fig (ii).

The magnitude of the flux due to each turn of the coil is

$$\Phi = BA\cos\theta$$

$$\Phi = BA\cos wt$$

For N number of turns,

$$\Phi = N\,BA\cos wt \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(i)$$

From Faraday's law of electromagnetic induction, the induced emf is equal to the rate of change of flux.

$$\text{i.e} \quad E = -\frac{d\phi}{dt}$$

$$\text{i.e} \quad E = -\frac{d(N\,BA\cos wt)}{dt}$$

$$= -NBA\frac{d(\cos wt)}{dt}$$

$$= -NBA\,(\sin wt).\,w$$

$$\therefore E = NBAw\sin wt \ldots\ldots\ldots\ldots\ldots\ldots(ii)$$

The emf will be maximum when wt= 90^0

$$\text{maximum emf } (E_o) = NBAw \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(iii)$$

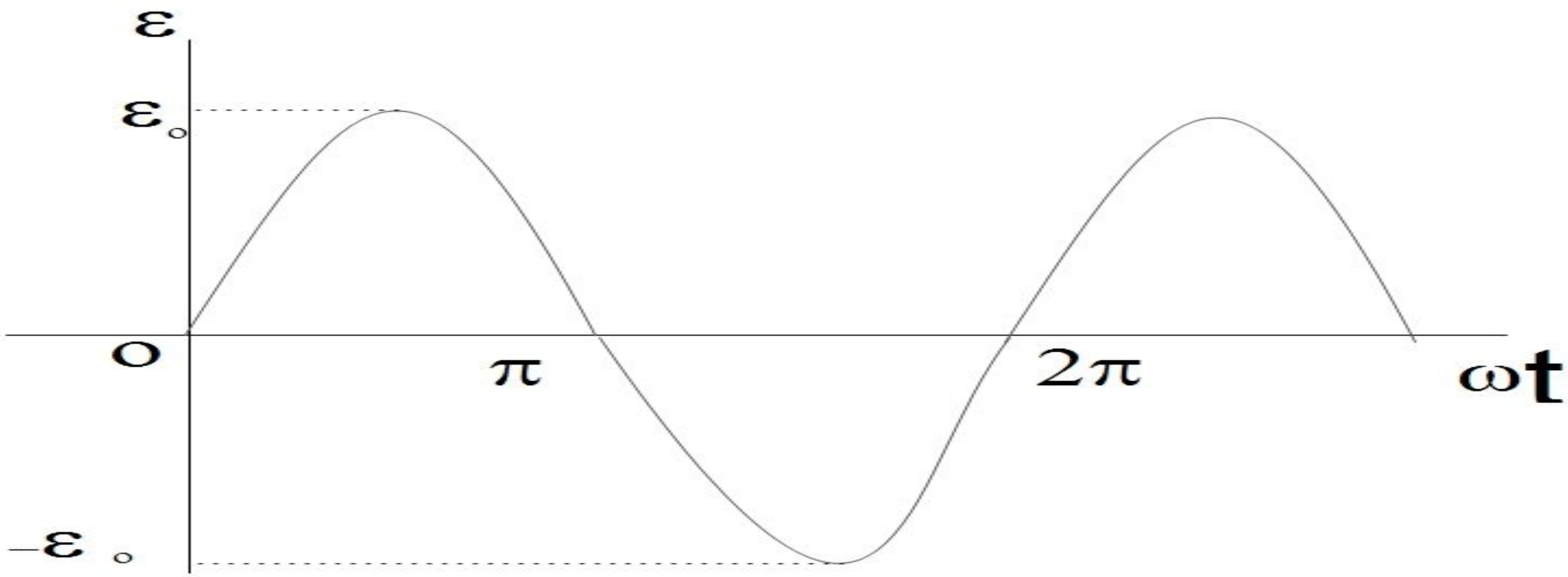
The eqn (ii) becomes

$$E = E_o \sin wt \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(iv)$$

This emf is called alternating emf

The eqn (iv) is the required expression for induced emf in a coil rotating in a magnetic field.

- Graphically,



The current in the coil is given by

$$I = \frac{E}{R}$$

$$I = \frac{E_o \sin wt}{R}$$

$$I = I_o \sin wt$$

where $I_o = \frac{E_o}{R}$ is the maximum value of the induced current.

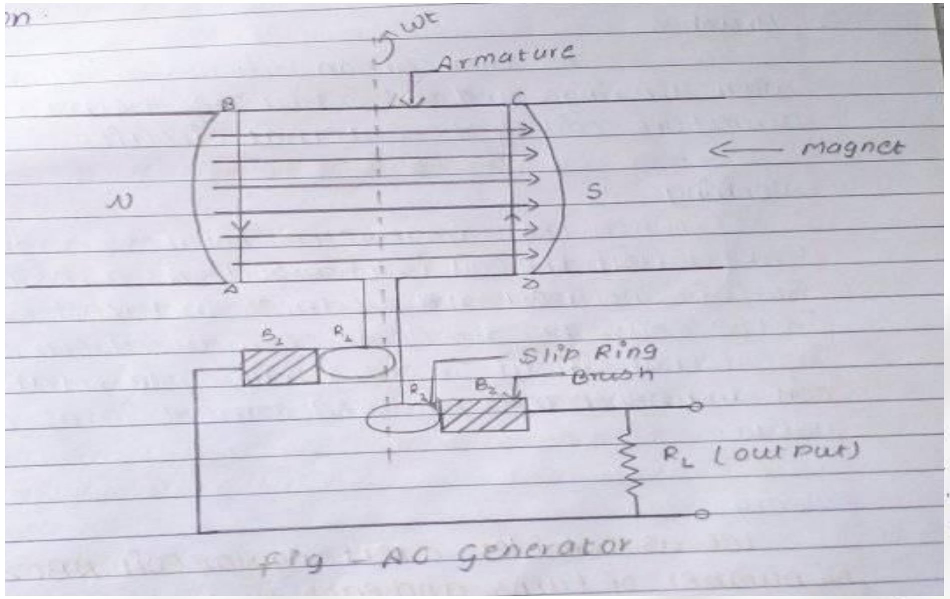
- 4. A wire 30cm long is moved in a uniform magnetic field having 2x 10^{-4} T. If the induced emf in a wire is 1.25 10^{-2} V. find the velocity and direction of the wire.

AC Generator

A device which is used to convert mechanical energy into electrical energy in the form of alternating current is called ac generator.

Principle :

It works on the principle of electromagnetic induction. i.e. when the coil rotates in a magnetic field, emf is induced in it.



Construction

It consists of following parts

i. Armature:

A rectangular coil ABCD consists of large number of turns of insulated copper wire wound over soft iron core is called armature.

ii. **Field magnet**: The uniform magnetic field is generated by two strong permanent magnet 'N' and 'S'.

iii. **Slip rings**: The two ends of the armature are connected with the rings R_1 and R_2. These rings are rotated with the rotation of coil.

iv. **Carbon brushes**: The carbon brushes B_1 and B_2 slide along the rings and connected to the external electrical circuit.

Working :

When the armature of the generator rotates in between two poles of the permanent magnet, magnetic flux linked with the armature is changed. As a result, an emf is induced at its end and produces induced current. The direction of induced current changes in the external circuit after every half rotation of armature. This process is repeated and hence the produced current is alternating nature.

Theory :

Let us consider a rectangular coil having 'N' number of turns and each having area 'A' is placed in a uniform magnetic field 'B'. suppose the plane of the coil is perpendicular to the magnetic field. At any instant, the coil is rotated anticlockwise direction by an angle θ with the direction of magnetic field as shown in above figure.

The magnitude of the flux due to each turn of the coil is

$$\Phi = BA\cos\theta$$

$$\Phi = BA\cos wt$$

For N number of turns,

$$\Phi = N\,BA\cos wt \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(i)$$

From Faraday's law of electromagnetic induction, the induced emf is equal to the rate of change of flux.

$$\text{i.e}\quad E = -\frac{d\phi}{dt}$$

$$\text{or,}\quad E = -\frac{d(N\,BA\cos wt)}{dt}$$

$$= -NBA\frac{d(\cos wt)}{dt}$$

$$= -NBA(-\sin wt).\,w$$

$$\therefore E = NBAw\sin wt \ldots\ldots\ldots\ldots\ldots\ldots(ii)$$

The emf will be maximum when wt= 90^0

$$\text{maximum emf } (E_o) = NBAw \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(iii)$$

The eqn (ii) becomes

$$E = E_0\sin wt \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(iv)$$

This emf is called alternating emf

Special cases :

i) when $wt=0$, $E=0$

ii) When $wt=\frac{\pi}{2}$, $E=E_o$

iii) When $wt=\pi$, $E=0$

iv) When $wt=\frac{3\pi}{2}$, $E=-E_o$

v) When $wt=2\pi$, $E=0$

If we plot the graph between E and wt, the graph obtained is sinusoidal curve as shown in below figure

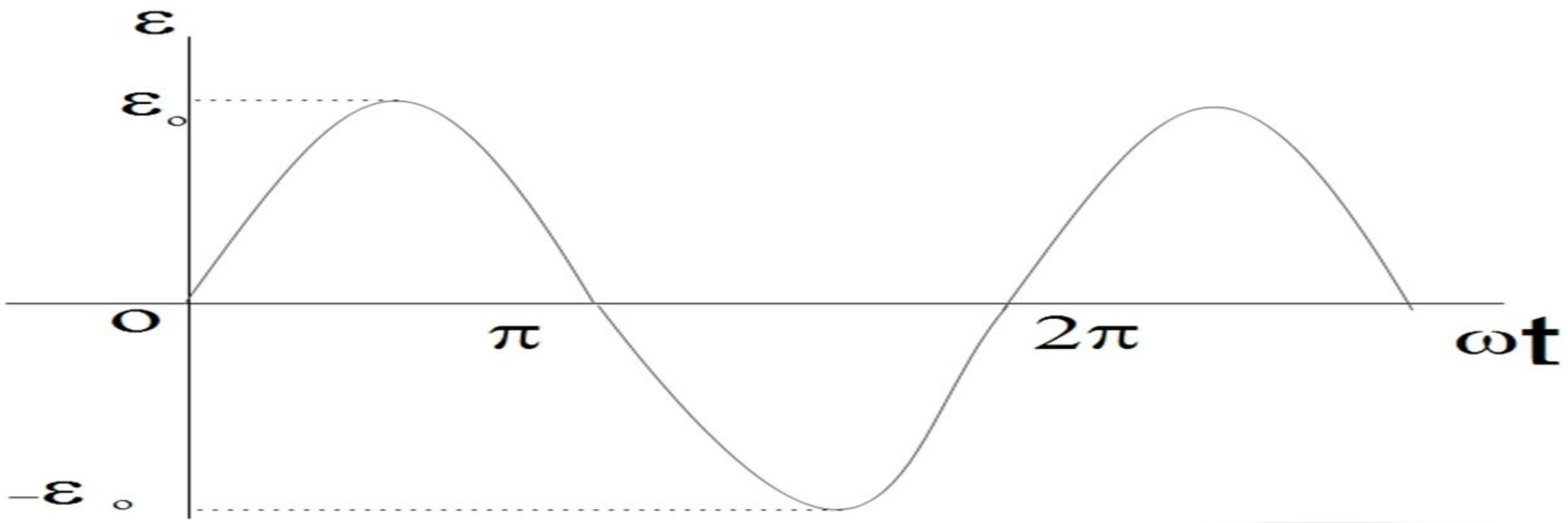


Fig: Variation of E and *wt*

Again,

The instantaneous current in the coil is given by

$$I = \frac{E}{R}$$

$$I = \frac{E_o \sin wt}{R}$$

$$I = I_o \sin wt$$

where $I_o = \frac{E_o}{R}$ is the maximum value of the induced current.

MCQ:

1. When a coil rotated in magnetic field the induced current in it

i. remains same

ii. becomes zero

iii. becomes maximum

iv. continuously changes

2. An electric generator converts

1.Electric energy into light energy
2.Mechanical energy into electric energy
3.Electric energy into sound energy
4.Electric energy into mechanical energy

3. The basic principle on which the AC generator works is

1.Electromagnetic induction
2.Energy conservation
3.Lenz law
4.Momentum conservation

Numerical

The armature of a small generator consists of a flat square coil with 120 turns and sides with a length of 1.6cm. The coil rotates in a magnetic field of 0.75T. What is the average speed of the coil if the maximum emf produced is 24mv?

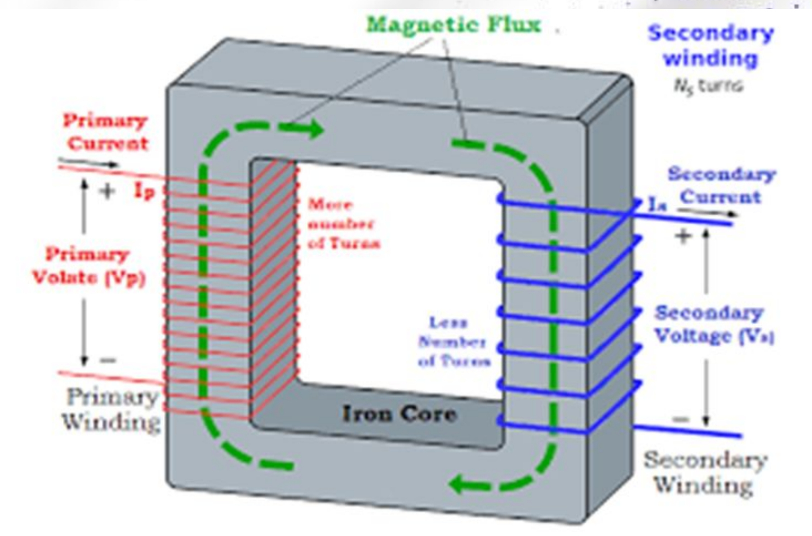
Eddy current (Foucault's current)

- The induced circulating current produced in a metal itself due to change in magnetic flux linked with the metal are called eddy current. The current induced not only in conducting wire or coil, but also in a conducting sheet or metallic piece placed on it.
- The current induced in a conducting sheet or metallic piece due to change in magnetic flux linked with it is called eddy current (Foucault's current).
- Eddy current produces heat in the conducting sheet or metallic piece due to which there is a loss of power.
- To minimize the energy loss due to eddy current, the iron core should be laminated by thin metal sheet or varnish.
- The direction of eddy current is given by Lenz's law.

$$\text{Eddy current (I)} = \frac{\text{induced emf}}{\text{resistance of conducting sheet}} = \frac{E}{R}$$

$$\text{Since , } E = -\frac{d\Phi}{dt}$$

$$\therefore I = -\frac{\frac{d\Phi}{dt}}{R}$$

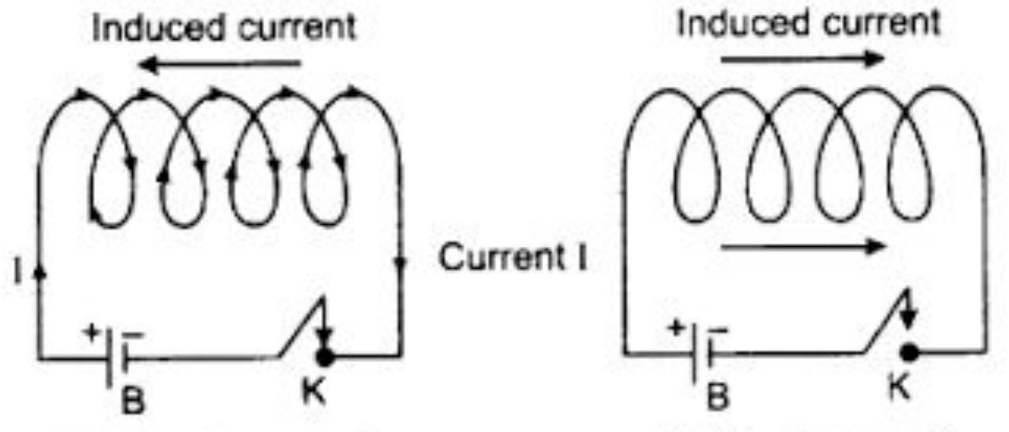


Uses of Eddy currents (Foucault's currents)

1. Electromagnetic brakes (control the speed of electric trains)
2. Induction furnace
3. Speedometer (measure instantaneous speed of vehicle)
4. Coating thickness measurement
5. Heating of tissue in human body

Self Induction

The change of the magnetic flux linked with the coil during growth and decay of the current in the coil produces the induced emf in the same coil. This phenomena is called self induction.



(a) Key is pressed (b) Key is opened

Consider a coil connected with a battery and a key.

When the key is pressed(switch on), the current starts growing through the coil, as a result the flux linked with the coil also starts growing through it. This produces induced emf in the coil. According to the lenz law, the direction of induced emf is such that it opposes the growth of the current in the circuit as shown in fig (a).

When the key is opened(off), the current in the circuit starts decaying as a result flux linked with the coil also decreasing which in turn produces induced emf in the coil. This also opposes decay of current as shown in fig (b). An opposing induced emf is produced in the coil during both growth and decay of current.

Coefficient of self induction or Self inductance

The property of the coil by virtue of which it opposes the growth or decay of the current flowing through it is called Self inductance.

We know, flux linked with the coil is directly proportional to the current flowing through a coil.

$$\Phi \propto I$$

$$\Phi = LI \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}$$

Where L is proportionality constant called self inductance.

From faraday's law of electromagnetic induction, the induced emf in a coil is

$$E = -\frac{d\phi}{dt}$$

$$= -\frac{d(LI)}{dt}$$

$$= -L\frac{dI}{dt}$$

$$L = -\frac{E}{\frac{dI}{dt}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(ii)}$$

- If $\frac{dI}{dt} = 1A/s$

  then L= - E

Here, self inductance can also be defined as the induced back emf per unit rate of change of current. The SI unit of Self inductance is Henry(VSA^{-1}).

Q.N Define one henry?

Self inductance of solenoid

- Let us consider a solenoid of length l, cross-sectional area 'A' and total no. of turns 'N'. If I be the current flowing through the coil then magnetic field inside the solenoid is

$$B = \mu_o nI$$

$$= \mu_o \frac{N}{l} I \ldots\ldots\ldots\ldots\ldots(i)$$

For N no. of turns, flux linked with the solenoid is

$$\Phi = NBA \ldots\ldots\ldots\ldots\ldots(ii)$$

from eqn (i) and (ii)

$$\Phi = \mu_o \frac{N}{l} I \times NA$$

$$\Phi = \mu_o \frac{N^2}{l} AI \ldots\ldots\ldots\ldots(iii)$$

Also, magnetic flux linked with the coil is,

$$\Phi = LI \ldots\ldots\ldots\ldots\ldots\ldots\ldots(iv)$$

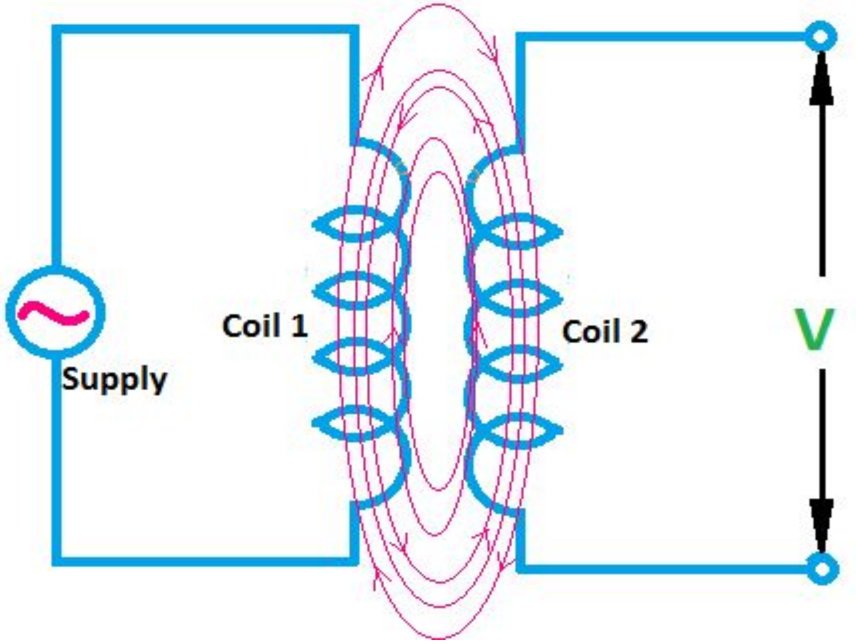
from eqn (iii) and (iv)

$$L = \frac{\mu_o N^2 A}{l}$$

This is the required expression for self inductance of solenoid.

Mutual induction

- The phenomenon of production of induced emf in a secondary coil due to change in current in primary coil is called mutual induct



We know, the magnetic flux linked with the secondary coil is directly proportional to the current flowing through the primary coil. i.e

$$\Phi_s = M\, I_p$$

Where M is proportionality constant called mutual inductance.

From faraday's law of electromagnetic induction, the induced emf in a secondary coil is

$$E_s = -\frac{d\Phi_s}{dt}$$

$$= -\frac{d(MIp)}{dt}$$

$$\therefore E_s = M\frac{dI_p}{dt} \quad \text{(Taking magnitude only)}$$

or, $$M = \frac{E_s}{dI_p/dt}$$

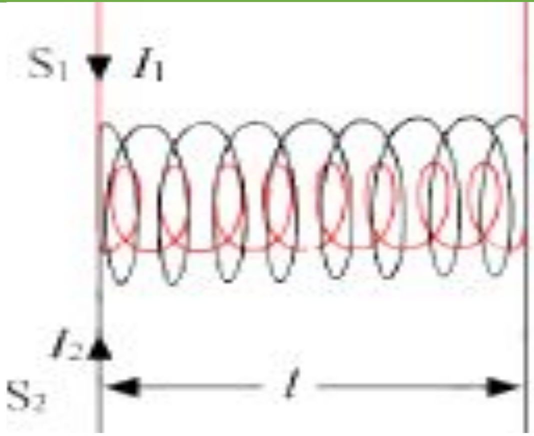
If $\frac{dIp}{dt} = 1As^{-1}$ then ,

$$\therefore M = E_s$$

Mutual inductance is also defined as the induced emf produced in secondary coil due to unit rate of change of current in primary coil.

Its SI unit is Henry

Mutual induction between two co-axial solenoids



Let us consider two solenoids S_1 and S_2 having N_1 and N_2 no. of turns respectively. Let 'l' be the equal length of co-axial solenoid. Suppose I_1 be the current flowing through solenoid S_1 as shown in figure.

We know, the flux linked with S_2 is directly proportional to the current flowing in S_1 solenoid. i.e

$$\Phi_2 \; \alpha \; I_1$$

$$\Phi_2 = M_{12} I_1 \; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

where M_{12} is the mutual inductance of S_1 w.r.t S_2

The magnetic flux linked with S_2 due to N_2 no. of turns is

$$\Phi_2 = N_2 B_1 A \qquad (2)$$

The magnetic field inside the solenoid S_1 is given by

$$B_1 = \mu_o n_1 I_1$$

$$B_1 = \frac{\mu_o N_1 I_1}{l} \qquad (3)$$

Using eq^n (3) and (2)

$$\Phi_2 = \frac{\mu_o N_1 N_2 A}{l} I_1 \qquad (4)$$

comparing eq^n (1) and (4)

$$M_{12} = \frac{\mu_o N_1 N_2 A}{l}$$

Similarly,

$$M_{21} = \frac{\mu_o N_1 N_2 A}{l}$$

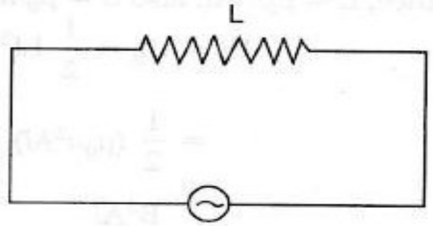
Therefore, the mutual induction between two coils is same as it doesn't matter which of the two coils carries the current.

$$M = M_{12} = M_{21} = \frac{u_o N_1 N_2 A}{l}$$

$$\therefore \; M = \frac{u_o N_1 N_2 A}{l}$$

Question: Two plane coils having number of turns 1000 and 2000, and radii 5cm and 10 cm respectively are placed co-axially in the same plane. Calculate their mutual inductance.($u_0 = 4\pi x 10^{-7} H m^{-1}$)

Energy stored in an inductor



Consider an inductor of inductance 'L' having initially zero current in the circuit. Let I be the current at any instant of time then the emf developed in the inductor is

$$E = -\frac{LdI}{dt} \qquad (1)$$

The power supplied to the inductor is

$$P = EI \qquad (2)$$

Combining (1) and (2)

$$P = \frac{LIdI}{dt} \quad \text{(taking magnitude only)}$$

The small amount of work done (energy supplied) in small time dt is

$$dw = Pdt$$

$$= L\,I\,\frac{dI}{dt}\,dt$$

$$\therefore dw = LI\,dI$$

The total work done is obtained by integrating dw from 0 to I

$$W = \int_0^I dw$$

$$= \int_0^I LIdI$$

$$\therefore W = \frac{LI^2}{2}$$

This work done will be stored in the form of magnetic energy in the inductor

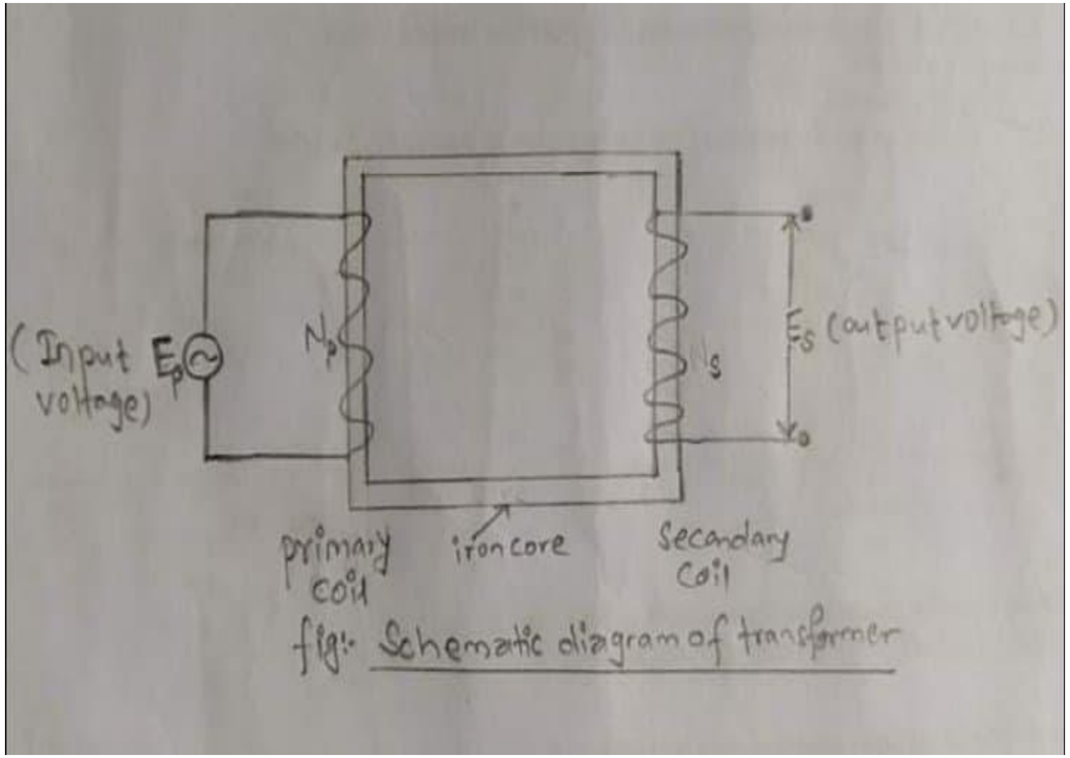
Therefore, the energy stored in the inductor is

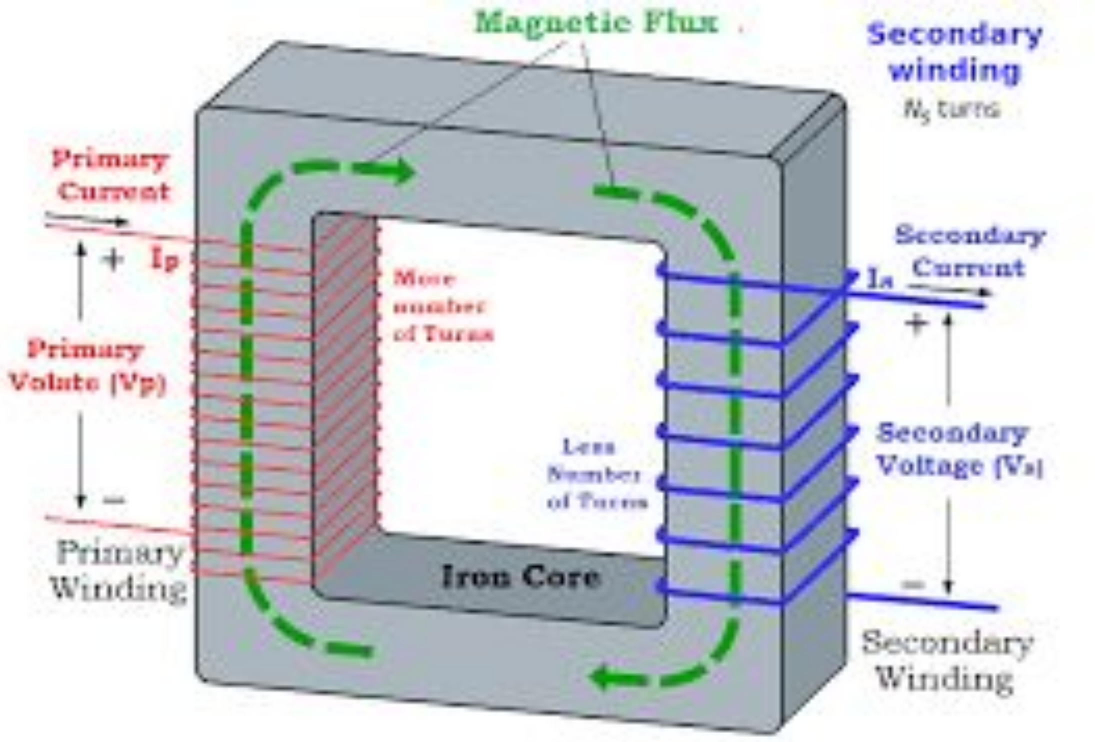
$$U = \frac{1}{2} LI^2$$

Transformer

It is a device which is used to step up or step down the input voltage.

Principle: It is based on the principle of mutual induction.





Construction:

It consists of two coils primary and secondary wounded over common laminated iron core. If the no. of turns in primary coil is greater than that of secondary coil then such type of transformer is called step down transformer. If the no. of turns in secondary coil is greater than that of primary coil then such type of transformer is called step up transformer.

Working:

when an ac is supplied to the primary coil, magnetic flux linked with the coil is changed. As a result an emf is induced in primary coil due to self induction and hence emf is produced in secondary coil due to mutual induction.

Theory:

If N_P and N_S be the no. of turns in primary coil and secondary coil with input voltage E_P and output voltage E_S, then

From faraday's law of electromagnetic induction, the induced emf in a primary coil is

$$E_P = -N_P \frac{d\Phi}{dt} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

The induced emf in a secondary coil is

$$E_S = - N_S \frac{d\Phi}{dt} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Dividing eq^n (2) and (1)

$$\frac{E_S}{E_P} = \frac{N_S}{N_P}$$

$$\frac{E_S}{E_P} = \frac{N_S}{N_P} = K$$

Where, $\frac{N_S}{N_P} = K$ is called transformation ratio

For no loss of power,

Input power= output power

$$E_P I_P = E_S I_S$$

$$\frac{E_S}{E_P} = \frac{I_P}{I_S} = K$$

$$\frac{E_S}{E_P} = \frac{N_S}{N_P} = K$$

Special cases:

i) if $k>1$,
$N_S > N_P$
$E_S > E_P$
Hence the transformer is step up

ii) if $k<1$,
$N_S < N_P$
$E_S < E_P$
Hence the transformer is step down

Efficiency of transformer:

The efficiency of transformer is defined as the ratio of output power to the input power.

$$\text{Efficiency } (\eta) = \frac{\text{Out power}}{\text{Input power}} = \frac{E_S I_S}{E_P I_P}$$

$$\eta = \frac{\text{Out power}}{\text{Input power}} \times 100\%$$

Efficiency of transformer ranges from 90 % -99 %. It is not 100 % due to losses of energy.

A step down transformer transforms a supply line voltage 220 volts into 100 volts. Primary coil has 500 turns. The efficiency and power transmitted by the transformer are 80 % and 80kw. Find

a) The no. of turns in the secondary coil

b) Power supplied

Hints: $E_P = 220V$, $E_S = 100V$, $\eta = 80\%$, $N_P = 500$, Power transmitted(output power)=80kw

N_S = ?, power supplied(input power)= ?

Apply: a) $\frac{E_S}{E_P} = \frac{N_S}{N_P}$, b) $\eta = \frac{\text{Out power}}{\text{Input power}} \times 100\%$

Question: A transformer has 500 turns in the primary coil and 100 turns in the secondary coil. What is output voltage if the input voltage is 4000volts? If the transformer is assumed to have an efficiency of 100%, what primary current is required to draw 2000watts from the secondary?

Energy losses in transformer:

a) Copper loss:

The copper wires used in windings of the transformer leads to Joule heating effect ($E=I^2Rt$). Hence some energy is lost in the form of heat. To minimize this loss, thick wires considerably low resistance are used.

b) Flux loss:

There is no perfect coupling between primary and secondary coil. So, the leakage of flux takes place which results in loss of energy.

c) Eddy current loss:

Due variations of magnetic flux between two coils of transformer, an induced current is setup in an iron core called as eddy current. These current produces heat in the core due to which some power losses. It is minimized using laminated iron core.

d) **Hysteresis loss:**

This loss of energy is due to repeated magnetization and demagnetization of the iron core caused by the alternating input current. This energy loss can be minimized by using suitable material having narrow hysteresis loop for the core of the transformer.

Question: What are different loss power losses in a transformer? What measure do you take to minimize these losses?

Question: Mention two types of loss in a transformer.

Question: Why transformer cannot be used in dc circuits?

OR

Why can't a transformer be used to step up or step down the dc voltage?

OR

Explain why a transformer cannot work with direct current

b. What inductance would be needed to store **1**kwh of energy in a coil carrying a 200A current?

Energy strored (U)=1kwh= 1000x $\frac{J}{s}$ x60 x 60 s

1. What is meant by ideal transformer? Describe principle, construction and working of transformer. What are the uses of transformer?

2. Two closely wound circular coils have the same number of turns but one has twice the radius of the other. What is the ratio of self inductance of the two coils?

4. A 2.0 H solenoid is connected in series with a resistor, so that the total resistance is 0.50Ω to a 2.0V d.c supply. What is

a. the final current?

b. the initial rate of change of current with time?

c. the rate of change of current with time when the current is 2.0?

5. Why is the long distance transmission of a.c is economical?

6. Why do we prefer a choke coil to rheostat in an a.c circuit?

7. A coil of wire of certain radius has 600 turns and a self inductance of 108mH. What will be the self inductance of another similar coil with 500 turns?

8. A sheet of copper is placed between the poles of an electromagnet with the magnetic field perpendicular to the sheet. When it is pulled out, a considerable force is required and the force required increases with speed. Why?

9. If the no. of turns of solenoid is doubled, keeping the other factors constant, how does the self-inductance of the solenoid change?

10. How can an a.c. generator be converted into d.c. generator?

1. The role of the inductor is equivalent to

a. Inertia b. force c. energy d. momentum

2. The energy stored in a 50 mH inductor carrying a current of 4A is

a. 0.1J b. 0.4J c. 0.04J d. 0.01J

3. A dynamo

a. Creates mechanical energy b. creates electrical energy c. converts electrical to mechanical d. converts mechanical to electrical

4. If supply frequency of a transformer increases, the secondary output voltage of transformer

a. Increases b. decreases c. remain same d. both a and b

Hint: in transformer the input frequency and output frequency remain same.

5. The self inductance of a straight conductor is

a. Zero b. infinity c. negative d. positive

$N\Phi = LI$, for straight conductor N=0, so L=0

6. Eddy current does not cause

a. Damping b. heating c. sparking d. energy loss

In eddy current, the electrons are only at the surface of the conductor and the electrons do not jump off to enter the air and heat it. Hence, there is no sparking. Therefore, eddy currents do not cause sparking. So, the correct answer is "Option c".

6. Eddy current does not cause

a. Damping b. heating c. sparking d. energy loss

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7. Transformer cores are laminated in order to

a. Reduce eddy current b. increase electric flux c. reduce hysteresis loss d. reduce copper loss

8. A transformer

a. Changes ac to dc

b. Changes dc to ac

c. Up or down ac voltage

d. Up or down dc voltage

Numerical Problems

1. The magnetic flux passing perpendicular to the plane of coil is given by $\phi=4t^2+5t+2$ where 'ϕ' is in Weber and 't' is in second. Calculate the magnitude of instantaneous emf induced in the coil when t=2sec. Also find the current induced in the coil if the resistance of the coil is 3Ω. [21V, 7A]

2. A straight conductor of length 25cm is moving perpendicular to its length with a uniform speed of 10m/s making an angle of 45° with a uniform field of 10T. Calculate the emf induced across its length. [17.68V]

3. A coil of 100 turns, each of area $2\times 10^{-3}\ m^2$ has a resistance of 12Ω. It lies in a horizontal plane in a vertical magnetic flux density of 3×10^{-3} Wb m^{-2}. What charge circulates through the coil if its ends are short-circuited and the coil is rotated through 180° about a diametrical axis? [$10^{-4} C$]

4. A plane circular coil has 200 turns and its radius is 0.10m. It is connected to a battery. After switching on the circuit of 2A is set up in the coil. Calculate the energy stored in the coil. ($\mu_0=4\pi\times 10^{-7}$ H/m) [$1.58\times 10^{-2} J$]

7. A rectangular coil of 100 turns has dimensions 15 cm × 10 cm . It is rotated at the rate of 300 revolutions per minute in a uniform magnetic field of flux density 0.6T. Calculate the maximum emf induced in it [28.27V]

6. A long solenoid of 1000 turns and cross sectional area 2× 10^{-3} m^2 carries a current of 2A and produces a flux density 52× 10^{-3} T inside it. Calculate the self inductance of the coil. [0.052H]

5. An air cored solenoid having a diameter of 4cm and a length of 60cm is wound with 4000 turns. Find the inductance of the solenoid. What is the inductance of the solenoid if it has iron core of relative permeability 400? (μ_0=4π× 10^{-7} H/m) [0.0421H, 16.84H]

8. A long solenoid with 15 turns per cm has a small loop of area 2 cm^2 placed inside, normal to the axis of the solenoid. If the current by the solenoid changes steady from 2A to 4A in 0.1 second, what is the induced voltage in the loop, while the current is changing? [7.5×10^{-6}V]

9. A long solenoid of 1000 turns and cross sectional area $2 \times 10^{-2} m^2$ carries a current of 2A and produces a flux density of $5 \times 10^{-3} T$ in the middle of the coil. Assuming the value of flux density at all sections of the solenoid, Calculate (a) its self inductance. (b) A small coil X is now placed in the middle of the solenoid so that the flux links its turns normal to X has 10 turns and mean area $4 \times 10^{-5} m^2$. Calculate the mutual inductance between the coil X and solenoid. [$5 \times 10^{-2} H, 10^{-6} H$]

10. Two plane coils having number of turns 1000 and 2000, and radii 5cm and 10cm respectively are placed co-axially in the same plane. Calculate their mutual inductance. (μ_0 =4π× 10^{-7} H/m) [0.0987H]

11. A transformer has 500 turns in the primary coil and 100 turns in the secondary coil. What is the output voltage if the input voltage is 4000 volts? If the transformer is assumed to have an efficiency of 100%, what primary current is required to draw 2000 watts from the secondary? [800V, 0.5A]
12. A step down transformer transforms a supply line voltage 220 volts into 100 volts. Primary coil has 500 turns. The efficiency and power transmitted by the transformer are 80% and 80 kw. Find (a) the number of turns in the secondary coil (b) power supplied. [250 turns, 100kw]
13. Find the emf induced in a straight conductor of length 25 cm, on the armature of a dynamo and 12 cm from the axis, when the conductor is moving in a uniform radial magnetic field of 0.5 T. The armature is rotating at 1000 revolutions per minute. [1.57 V]
14. When a wheel with metal spokes 1.2m long is rotated in a magnetic field of flux density 5×10^{-5} T normal to the plane of wheel, an emf of 10^{-2} V is induced between the rim and axle. Find the rate of rotation of the wheel. [44.2 rev/sec]
15. A circular metal disc of area $3.0\times 10^{-3} m^2$ is rotated at 50 rev/sec about an axis through its center perpendicular to its plane. The disc is in uniform field of flux density $5.0 \times 10^{-3} T$. In the direction of axle. What is the value of induced emf. [$7.5 \times 10^{-4} V$]


Thank
you