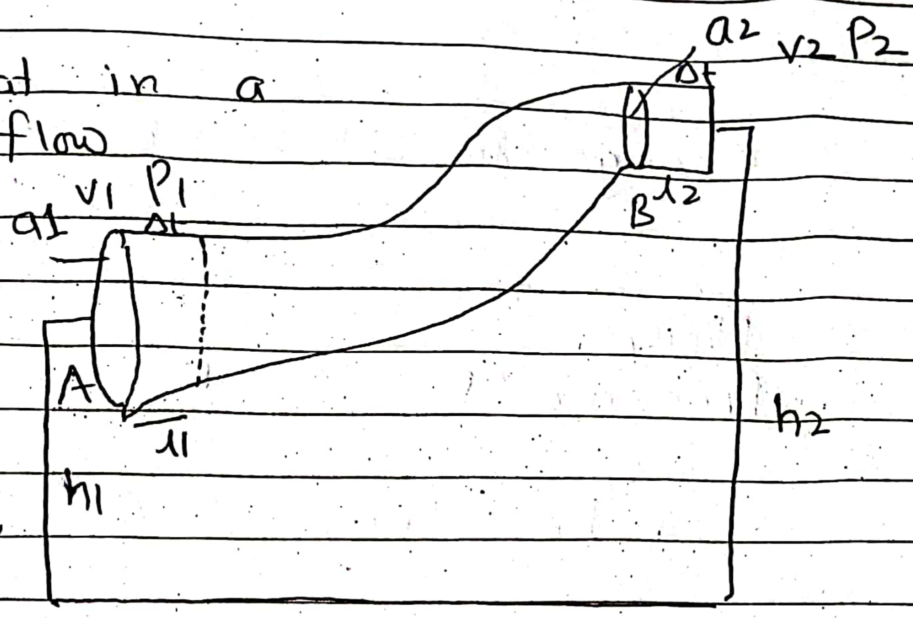
Bernoullis theorem

It states that in a stream line flow of an ideal fluid (Incompressible & non viscous), the Pressure energy, Kinetic energy, & Potential energy per unit mass remain constant



$$E = \frac{P}{\rho} + gh + \frac{1}{2}v^2 = \text{constant}$$

let a_1 and a_2 be cross section area at A & B
v_1 and v_2 be the velocity at A & B
P_1 and P_2 be the ~~cross sect~~ pressure at A & B
h_1 and h_2 be the height of A & B respectively

From work energy theorem
Total work done = change in PE + change in KE —(1)

Change in KE = $\frac{1}{2}dmv_2^2 - \frac{1}{2}dmv_1^2$ — (2)

Change in PE = $dmgh_2 - dmgh_1$ — (3)

Now,

Pressure $(P_1) = \frac{F_1}{a_1}$

$F_1 = P_1 a_1$ — (4)

work done ~~is~~ to move small amount dm through distance l_1

$$W_1 = F_1 \cdot l_1$$
$$= P_1 a_1 l_1$$
$$= P_1 a_1 v_1 \cdot \Delta t$$

$\left[\frac{dV}{dt} = av \right]$

$$= P_1 dV_1$$

$$= P_1 \frac{dm}{\rho} \quad \text{— (5)}$$

$$W_2 = P_2 \frac{dm}{\rho} \quad \text{— (6)}$$

Total work done = $W_1 - W_2$

$$= (P_1 - P_2) \frac{dm}{\rho} \quad \text{— (7)}$$

Now,

$$(P_1 - P_2)\frac{dm}{\rho} = dm\,g\,h_2 - dm\,h_1 + \frac{1}{2}dm\,v_2^2 - \frac{1}{2}dm\,v_1^2$$

$$\frac{P_1}{\rho} - \frac{P_2}{\rho} = gh_2 - gh_1 + \frac{1}{2}v_2^2 - \frac{1}{2}v_1^2$$

$$\frac{P_1}{\rho} + gh_1 + \frac{1}{2}v_1^2 = \frac{P_2}{\rho} + gh_2 + \frac{1}{2}v_2^2$$

$$\therefore \boxed{\frac{P}{\rho} + gh + \frac{1}{2}v^2 = \text{constant}}$$