Alternating Current

Numerical Problems:

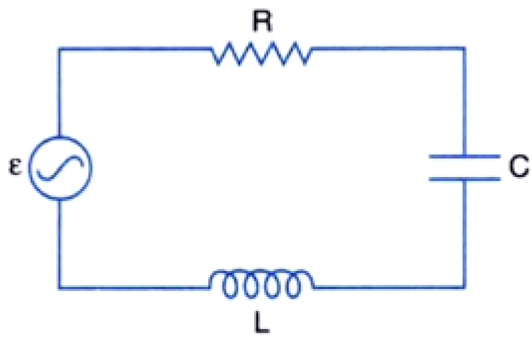
1. An iron cored coil of inductance 3H and 50 Ω resistance is placed in series with a resistor of 550 ohm, and a 100V, 50Hz ac supply is connected across the arrangements. Find the current following in the coil and the voltage across the coil. [0.09A, 84.8V]
2. An iron cored coil of inductance 2H and 50 Ω resistance is placed in series with a resistor of 450 ohm, 200v, 50Hz ac supply is connected across the arrangements. Find
   i) the current following in the coil [0.25A]
   ii) the voltage across the coil. [15.75V]
   iii) The phase angle relative to the voltage supply. [51 .3°]
3. A constant A.C. supply is connected to a series circuit consisting of a resistance of 300Ω in series with a capacitance 6.67 μF, the frequency of the supply being 3000/2 πHz. It is desired to reduce the current in the circuit to half its value. Show how this could be done by placing an additional resistance. [Resistance to be added = 306.1 ohm]
4. A circuit consists of capacitor of 10 μF and resistor of 1000 Ω. An alternating emf of 12V(rms) and frequency 50Hz is applied. Calculated the current flowing and voltage across the capacitor. [0.0114A, 3.63V]
5. A circuit consists of a capacitor of 2μF and a resistor of 1000Ω. An alternating emf of 12V and frequency 50Hz is applied. Find the voltage across the capacitor and the phase angle between the applied emf and the current. [10.2V, 57.9°]
6. An ac source of 220V, 50Hz is connected to series circuit containing a resistor R and inductor L and a capacitor C. If R=200Ω, L=0.5H and C=10μF, calculate, (i) the current in the circuit, [0.856A] (ii) the phase angle [-38.9°]and (iii) the power consumed in the circuit. [146.56W]
7. A coil of inductance 0.1 H and negligible resistance is in series with a resistance 40Ω. A supply voltage of 50V (rms) is connected to them, if the voltage across L is that across R, calculate the voltage across the inductor and frequency of the supply. [35.35V, 63.7Hz]
8. L-C-R alternating current series circuit of L=1H, C=1μF, and R=100Ω are connected in series with a source of frequency 50Hz. What is the phase shift between current and voltage? [88°, voltage lags behind current]
9. A coil having inductance and resistance is connected to an oscillator giving a fixed sinusoidal output voltage of 5V rms. With the oscillator set at a frequency of 50Hz, the rms current in the coil is 1A and at a frequency of 100Hz, the rms current is 0.625A. Determine the inductance of the coil. [0.0114H]
10. A 50V a.c. supply is connected to a resistor having resistance 50Ω, in series with a solenoid whose inductance is 0.25H. The potential difference between the ends of the resistor is 25V. Find the resistance of the wire of the solenoid. Take frequency of the ac source is 50Hz. [11.89ohm]
11. A 50V, 50Hz, a.c. supply is connected to a resistor, of resistance 40Ω, in series with a solenoid whose inductance is 0.20H. The p.d. between the ends of the resistor is found to be 20V. What is the resistance of the wire of the solenoid? (Assume π^2=10) [37.79 ohm]

12. A coil of inductance 0.1H and negligible resistance is in series with a resistance R. A supply voltage of 40V(rms) is connected to them. If a voltage across L is equal to that across R, calculate the voltage across R and the frequency of the supply? [28.3V, 63.7Hz]
13. In a series LCR circuit, R=25Ω, L=30mH and C=10μF and these elements are connected to 240 ac (rms) 50Hz source. Calculate the current in the circuit and voltmeter reading across a capacitor. [0.774A, 246.37V]
14. An inductor, a resistor and capacitor are connected in series across an a.c. circuit. A voltmeter reads 60V when connected across the inductor, 16V across the resistor and 30V across the capacitor:
    i. What will the voltmeter read when placed across the series circuit? [34V]
    ii. What is the power factor of the circuit? [0.47]
15. The maximum capacitance of a variable capacitor is 33pF. What should be the self-inductance to be connected to this capacitor for the natural frequency of the LC circuit to be 810 KHz. Corresponding to A.m. broadcast band of Radio Nepal? [1.17x 10^{-3}H]
16. An alternating voltage 10V (rms) and 4 kHz frequency is applied to a resistor of resistance 5Ω in the series with a capacitor of capacitance 10μF. Calculate the r.m.s. potential differences across the resistor and the capacitor. [7.8V, 6.2V]
17. Alternative voltage in an ac circuit is represented by $V=100\sqrt{2}\sin(100\pi)$ volts. Find its roots mean square value and the frequency. [50Hz]
18. A circuit consists an inductor of 200μ H and the resistance of 10Ω in series with a variable capacitor and a 0.10V (r.m.s.), 1.0 MHz supply.
    i. The capacitance to give resonance [1.26×10^{-10}F]
    ii. The quality factor of the circuit at resonance [126]
19. A 100V, 50Hz AC source is connected to an LCR circuit containing L= 8.1 mH, C=12.5 μ F and R=10Ω all connected in series. Find the potential difference across the resistor. [3.9V]
20. A coil of inductance 0.1H and negligible resistance is in series with a resistance 40Ω. A supply voltage of 50v (rms) is connected to them. If the voltage across L is equal to that across R, calculate the voltage across the inductor and frequency of the supply. [R=35.35volt, Frequency= 63.7Hz]
21. A.C. mains of 200 volts and 50Hz is joined to a circuit containing an inductance of 100mH and a resistance of 20Ω in series. Calculate the power consumed. [576.8 Watt]
22. An iron cored coil of inductance 2H and resistance 50Ω is connected in series with a resistor of 950Ω. A 220V, 50Hz ac supply is connected across the arrangement. Find the current flowing in the circuit and the voltage across the coil. [current= 0.186ampere and Voltage=116.87V]
23. A 100V, 50Hz A.C. sources are connected to an LCR circuit Containing L=8.1mH, C=12.5 μF and R= 100Ω all are connected in series. Find the p.d. across the resistor. [37V]
24. An LCR series circuit, with L=0.12H, C=7.3 μF and R= 240Ω, carries current of 0.45A with a frequency of 400Hz
    i. What are the phase angle and power factor? [phase angle= 45.57° and power factor= 0.70]

ii. What is impedance of the circuit? [344Ω]

25. A coil of inductance 0.5H and negligible resistance is in series with resistance of 40 Ω. A supply voltage of 40V (rms) is connected across them. If the voltage across the coil is equal to that across resistor, calculate the voltage across each component and frequency of the ac supply. [V across R= 35.35V, frequency= 12.74Hz]

26. Figure shows a series LCR circuit connected to a variable frequency 230 V source. L = 5.0 H, C = 80 μF, R = 40Ω.



(a) Determine the source frequency which drives the circuit in resonance. [1]

(b) Obtain the impedance of the circuit and the amplitude of current at the resonating frequency. [2]

(c) Determine the rms potential drops across the three elements of the circuit. [3]

(d) Show that the potential drop across the LC combination is zero at the resonating frequency. [1]

(e) How do you explain the observations that the algebraic sum of the voltages across the three elements is greater than the supplied voltages? [1]

WORKED OUT EXAMPLE

Example - 1. *An a.c. supply voltage is represented by the equation $v = 220\sqrt{2}\sin 100\pi t$. Determine (a) the peak voltage (b) the r.m.s. voltage (c) the angular frequency (d) the frequency (e) the period.*

Solution:

The given equation $V = 220\sqrt{2}\sin 100\pi t$ .... (1)

The standard equation $V = V_m \sin \omega t$ .... (2)

Comparing equations (1) and (2) we get,

the peak voltage $(V_m) = 220\sqrt{2}$ V

$$V_{r.m.s} = \frac{V_m}{\sqrt{2}} = \frac{220\sqrt{2}}{\sqrt{2}} = 220 \text{ V}$$

The angular frequency $(\omega) = 100\pi$

$$\text{Frequency (f)} = \frac{\omega}{2\pi} = \frac{100\pi}{2\pi} = 50 \text{ Hz}$$

$$\text{Time period (T)} = \frac{1}{f} = \frac{1}{50} = 0.02 \text{ s}$$

Example - 2. *An a.c. supply of frequency 50 Hz and voltage of 10 r.m.s is applied to (i) 5 Ω resistor (ii) 2H inductor and (iii) 1 μF capacitor. Determine the r.m.s. current flowing in each case.*

Solution:

i) R.m.s. voltage $(V_{rms}) = 10$V

Resistance $(R) = 5\ \Omega$

$$\text{R.m.s. current } (I_{rms}) = \frac{V_{rms}}{R} = \frac{10}{5} = 2\text{A}$$

ii) Inductance $(L) = 2$H,

iii) Capacitance $(C) = 1\ \mu F = 10^{-6}$ F

$$I_{rms} = \frac{V_{rms}}{X_C} = \frac{V_{rms}}{\frac{1}{2\pi fC}} = V_{rms} \times 2\pi fC$$

$$= 10 \times 2\pi \times 50 \times 10^{-6} = 0.0031 \text{ A}$$

Example - 3. *A circuit consisting of series arrangement of a resistor R of 20 Ω, an inductor L of 0.15 H and a capacitor C of 500 μF. If an alternating current of 0.2 A (rms) and frequency $\frac{100}{2\pi}$ Hz flows through the circuit, calculate the a.c. voltage (i) across each component, (ii) across R and L together (iii) across L and C together, (iv) the total voltage across L.C.R. What power is dissipated in each component?*

Solution:

Here, Current (I) = 0.2 A

Frequency $(f) = \frac{100}{2\pi}$ Hz

Resistance $(R) = 20\ \Omega$

Inductance $(L) = 0.15$ H,

Capacitance $(C) = 500\ \mu F = 500 \times 10^{-6}$ F

i) Voltage across R $(V_R) = IR = 0.2 \times 20 = 4.0$ V

$$\text{Voltage across, } (V_L) = IX_L = I \times 2\pi fL = 0.2 \times 2\pi \times \frac{100}{2\pi} \times 0.15 = 3\text{V}$$

$$\text{Voltage across C } (V_C) = IX_C = I\frac{1}{2\pi fC} = 0.2 \times \frac{1}{2\pi \times \frac{100}{2\pi} \times 500 \times 10^{-6}}$$

$$= 0.2 \times \frac{1}{0.05} = 4\text{V}$$

ii) Across R and L together

$$V^2 = V_L^2 + V_R^2$$

$$\text{or, } V = \sqrt{V_L^2 + V_R^2} = \sqrt{(3)^2 + (4)^2} = 5\text{V}$$

iii) Across L and C, $V = V_C - V_L = 4 - 3 = 1$ V

iv) Across L, C, R: $V = \sqrt{V_R^2 + (V_C - V_L)^2} = \sqrt{(4)^2 + (1)^2} = 4.1$ V

∵ L and C consumes no power. The power is dissipated only in the resistor.

So, power dissipated in the resistor

$$P = I^2 R = (0.2)^2 \times 20 = 0.8 \text{ W}$$

Example - 4. *A coil inductance L and negligible resistance is in series with a resistance R. A supply voltage of 40 V (r.m.s.) is connected to them. If the voltage across L is equal to that across R, calculate (a) the voltage across each component (b) the frequency f of the supply (c) the power absorbed in the circuit if L = 0.1 H, R = 40 Ω*

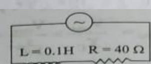
Solution:

Here,

Inductance (L) = 0.1 H

Resistance (R) = 40 Ω

Voltage (V) = 40 V



a) In LR circuit,

$$V_R^2 + V_L^2 = V^2$$

$$\because V_R = V_L$$

$$V_R^2 + V_R^2 = (40)^2$$

$$V_R^2 = \frac{(40)^2}{2} \text{ or } V_R = \frac{40}{\sqrt{2}}$$

or, Voltage across resistor $(V_R) = 20\sqrt{2} = 28.3$ V

∴ Voltage across inductor $(V_L) = V_R = 28.3$ V

b) ∴ $V_L = V_R$

$IX_L = IR$ where I is the current in the circuit

or, $X_L = R$ or $2\pi fL = R$

$$\text{Frequency (f)} = \frac{R}{2\pi L} = \frac{40}{2\pi \times 0.1} = 63.7 \text{ Hz.}$$

c) Power absorbed in the circuit = power absorbed in the resistor

$$= \frac{V_R^2}{R} = \frac{(20\sqrt{2})^2}{40} = \frac{20 \times 20 \times 2}{40} = 20 \text{ W}$$

Example - 5. *A.C. mains of 220 volts, 50 cycles is joined to a circuit containing an inductance of 100 mH and a resistance of 20 ohms in series. Calculate the power consumed.*

Solution:

Here,

Voltage (V) = 220 V

Frequency (f) = 50 cycles

Inductance (L) = 100mH = 0.1 H

Resistance (R) = 20 Ω

$$\text{Impedance (Z)} = \sqrt{R^2 + (X_L)^2} = \sqrt{R^2 + (2\pi fL)^2}$$

$$= \sqrt{(20)^2 + (2\pi \times 50 \times 0.1)^2} = 37.24\ \Omega$$

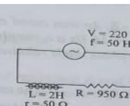
$$\text{Current (I)} = \frac{V}{Z} = \frac{220}{37.24} = 5.9 \text{ A}$$

$$\text{Power consumed } p = I^2 R = (5.9)^2 \times 20 = 698 \text{ W}$$

Example - 6. *An iron cored coil of 2 H and 50 Ω resistance is placed in series with a resistor of 950 Ω and a 220V, 50 Hz a.c. supply is connected across the arrangement. Find (i) the current flowing in the circuit (ii) the voltage across the coil.*

Solution:

Here, in LR circuit as shown in the figure,



Total resistance (R) = 950 + 50 = 1000 Ω

$$\text{Impedance (Z)} = \sqrt{R^2 + X_L^2}$$

$$= \sqrt{R^2 + (2\pi fL)^2}$$

$$= \sqrt{(1000)^2 + (2\pi \times 50 \times 2)^2} = 1181\ \Omega$$

$$\text{Current (I)} = \frac{V}{Z} = \frac{220}{1181} = 0.186 \text{ A}$$

The voltage across the coil

$$= IZ_L = I\sqrt{r^2 + X_L^2} = I\sqrt{r^2 + (2\pi fL)^2}$$

$$= 0.186\sqrt{(50)^2 + (2\pi \times 50 \times 2)^2} = 117.24 \text{ V}$$

Example - 7. *A 50 V, 50 Hz a.c. supply is connected to a resistor, of resistance 40 Ω in series with a solenoid whose inductance is 0.2 H. The p.d. between the ends of the resistor is found to be 20 V. What is the resistance of the wire of the solenoid? (Given $\pi^2 = 10$)*

Solution:

Here,

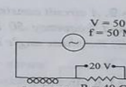
Voltage (V) = 50V

Frequency (f) = 50 Hz,

Inductance (L) = 0.2 H

Resistance (R) = 40 Ω

Voltage across (V_R) = 20 V



Let r be the resistance of the solenoid. The impedance of the circuit, Z.

$$Z = \sqrt{(R + r)^2 + X_L^2} = \sqrt{(40 + r)^2 + 4\pi^2 f^2 L^2}$$

$$= \sqrt{(40 + r)^2 + 4 \times 10 \times 2500 \times 0.04} = \sqrt{(40 + r)^2 + 4000}$$

$$\text{The current through the circuit} = \frac{V_R}{R} = \frac{20}{40} = 0.5 \text{ A}$$

$$\text{Impedance of the circuit} = \frac{V}{I} = \frac{50}{0.5} = 100\ \Omega$$

$$\therefore \sqrt{(40 + r)^2 + 4000} = 100$$

or, $(40 + r)^2 + 4000 = 100$

or, $(40 + r)^2 = 10{,}000 - 4000$

or, $(40 + r)^2 = 6000$ or $40 + r = 77.5$

or, $r = 37.5\ \Omega$

Hence, the resistance of the wire of the solenoid is 37.5 Ω.

Example - 8. *An iron cored coil of 2 H and 50 Ω resistance placed in series with a resistor of 450 Ω and a 100 V, 50 Hz a.c. supply is connected across the arrangement find (a) the current flowing in the coil, (b) its phase angle relative to the voltage supply, (c) the voltage across the coil.*

Solution:

(a) The reactance $X_L = 2\pi f L = 2\pi \times 50 \times 2 = 628\ \Omega$

Total resistance $R = 50 + 450 = 500\Omega$

Circuit impedance $Z = \sqrt{X_L^2 + R^2} = \sqrt{628^2 + 500^2} = 803\ \Omega$

$\therefore I = \dfrac{V}{Z} = \dfrac{100}{803}\ A = 0.124\ A$

(b) $\tan\theta = \dfrac{X_L}{R} = \dfrac{628}{500} = 1.256$

So $\theta = 51.5°$

(c) For the coil, $X_L = 628\ \Omega$ and $R = 50\ \Omega$

So coil impedance $Z = \sqrt{X_L^2 + R^2} = \sqrt{628^2 + 50^2} = 630\ \Omega$

Thus voltage across coil $V = IZ = 0.124 \times 10^{-3} \times 630 = 78.12\ V.$

Example - 9. *A circuit consists of capacitor of 2 μF and a resistor of 1000 Ω. An alternating e.m.f. of 12 V (r.m.s.) and frequency 50 Hz is applied. Find (1) the current flowing, (2) the voltage across the capacitor, (3) the phase angle between the applied e.m.f. and current, (4) the average power supplied.*

Solution:

In a CR circuit,

The reactance X_C of the capacitor is given by

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 2 \times 10^{-6}}$$

$$= 1592.35\ \Omega$$

$\therefore$ Total impedance $Z = \sqrt{X_c^2 + R_c^2} = \sqrt{1000^2 + (1592.35)^2}$

$= 1880.31\ \Omega$

(1) Current, $I = \dfrac{V}{Z} = \dfrac{12}{1880.31} = 6.4 \times 10^{-3} A$

(2) Voltage across C, $V_C = IX_C = \dfrac{12}{1880.31} \times 1592.35 = 10.2\ V$

(3) The phase angle θ is given by

$$\tan\theta = \frac{X_C}{R} = \frac{1592.35}{1000} = 1.59$$

(4) Average power supplied $= I^2R = \left(\dfrac{12}{1880.31}\right)^2 \times 1000 = 0.0407\ W.$

Example - 10. *A capacitor of capacitance C, a coil of inductance L and resistance R and a lamp are placed in series with an alternating voltage V. Its frequency f is varies from a low to a high value while the magnitude of V is kept constant. Describe and explain how the brightness of the lamp varies.*

If V = 0.01 V (r.m.s.) and C = 0.4 μF, L = 0.4 H, R = 10Ω calculate (i) the resonant frequency, (ii) the maximum current, (iii) the voltage across C at resonance, neglecting the lamp resistance.

Solution:

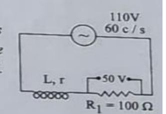
When f is varied, the impedance Z of the circuit decreases to a minimum value (resonance) and then increase in Z is a minimum when $X_L = X_C$, so that $Z = R$ at resonance. Since the current flowing in the circuit increases to a maximum and then decreases, the brightness of the lamp increases to a maximum at resonance and then decreases.

(i) Resonant frequency $f_0 = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{0.4 \times 0.4 \times 10^{-6}}} = \dfrac{10^3}{2\pi \times 0.4} = 396.08\ Hz$

(ii) Maximum current $= \dfrac{V}{R} = \dfrac{0.01}{10} = 0.001\ A$ (r.m.s.)

(iii) Voltage across $C = IX_C = 0.001 \times \dfrac{1}{2\pi \times 398 \times 0.4 \times 10^{-6}} = 1\ V$

Example - 11. *A 110 V, 60 Hz a.c. generator was connected to a series combination 100 Ω resistor and an inductor as shown in figure. The voltage across the resistor is 50 V and voltage is found to be leading the current by 45°. What are the resistance and inductance of the inductor?*

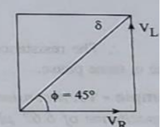


Solution:

The current $I = \dfrac{\text{Voltage}}{R_1} = \dfrac{50}{100} = 0.5\ A$

Impedance of the circuit $Z = \dfrac{V}{I} = \dfrac{110}{0.5} = 220\ \Omega$

But, $\tan\phi = \dfrac{V_L}{V_R} = \dfrac{X_L}{I(R_1 + r)} = \dfrac{X_L}{R_1 + r}$



or, $\tan 45° = \dfrac{X_L}{R_1 + r}$

or, $X_L = R_1 + r$ ... (i)

Now, $Z = \sqrt{(R_1 + r)^2 + X_L^2}$

or, $220 = \sqrt{(R_1 + r)^2 + (R_1 + r)^2}$

$\therefore (R_1 + r)^2 = \dfrac{220 \times 220}{2} = 220 \times 110$

$\therefore R_1 + r = \sqrt{220 \times 110}$

or, $100 + r = 155.56$

or, $r = 55.56\ \Omega$ .... (2)

Hence, $X_L = 100 + 55.56 = 155.56$

or, $2\pi f L = 155.56$

$\therefore L = \dfrac{155.56}{2\pi f} = \dfrac{155.56}{2\pi \times 60} = 0.413\ H = 413\ mH.$

Example - 12. *A supply of 240 V and 50 Hz provides a current of 1.00 A to a coil of self inductance of 0.200 H and a resistance of 50 Ω and the current is in phase with the potential difference of the source. Find the values of the components that must be put in series with the coil.*

Solution:

Here,

Inductance (L) = 0.200 H

Resistance (R) = 50.0 Ω

Current (I) = 1.0 A

Voltage (V) = 240 V

Frequency (f) = 50 Hz

In a LC circuit,

When the current is in phase with the potential difference, then

$X = X_L - X_C = 0$

$Z = \sqrt{R^2 + X^2} = R$

But, $Z = \dfrac{V}{I} = \dfrac{240}{1} = 240\ \Omega$

$\therefore R = 240\ \Omega$

$\therefore$ Resistance to be added in series $= 240 - 50 = 190\ \Omega$

Again, for resonance, $X_L = X_C$

or, $2\pi f L = \dfrac{1}{2\pi f C}$

or, $C = \dfrac{1}{4\pi^2 f^2 L} = \dfrac{1}{4\pi^2 (50)^2 \times 0.2}$

$= 50.6 \times 10^{-6}\ F = 50.6\ \mu F$

$\therefore$ The resistance of 190 Ω and a capacitor of 50.6 μF are connected in series with the coil to make in same phase.

Example - 13. *A constant a.c. supply is connected to a series circuit consisting of a 300 Ω in series with a capacitance of 6.67 μF, the frequency of the supply being 3000/2π Hz. It is desired to reduce the current in the circuit to half its value. Show how this could be done by placing either (a) an additional resistance or (b) an inductance, in series. Calculate in each case the magnitude of the extra component.*

Solution:

Here, Resistance (R) = 300 Ω

Capacitance (C) = 6.67 μF

Frequency (f) $= \dfrac{3000}{2\pi}$ Hz

Capacitive reactance $(X_C) = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi \times \frac{3000}{2\pi} \times 6.67 \times 10^{-6}}$

$= \dfrac{1000}{3 \times 6.67} = 50\ \Omega$

Impedance $(Z) = \sqrt{R^2 + X_C^2} = \sqrt{(300)^2 + (50)^2}$

$= 304.1\ \Omega$

New impedance $(Z') = 2Z = 2 \times 304.1$

$= 608.2\ \Omega$

a) Let R_1 be the resistance to be added; then

$Z' = \sqrt{(R + R_1)^2 + X_C^2}$

or, $(608.2)^2 = (300 + R_1)^2 + 2500$

or, $(300 + R_1)^2 = (608.2)^2 - 2500$

$= 369907.2 - 2500$

$= 367407.2$

$300 + R_1 = \sqrt{367407.2} = 606.1$

$\therefore R_1 = 606.1 - 300 = 306.1\ \Omega$

b) Let L be the inductance to be placed in series, then

$Z' = \sqrt{R^2 + (X_L - X_C)^2}$

or, $(608.2)^2 = (300)^2 + (X_L - 50)^2$

or, $(X_L - 50)^2 = (608.2)^2 - (300)^2$

$= 369907.24 - 9000$

or, $(X_L - 50)^2 = 279907.24$

or, $X_L - 50 = \sqrt{279907.24} = 529.06 \approx 529$

or, $X_L = 529 + 50 = 579$

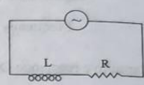
$L = \dfrac{579}{2\pi f}$

or, $L = \dfrac{579}{2\pi \times \frac{3000}{2\pi}} = 0.193\ H = 193\ mH$

Example - 14. *A coil having inductance and resistance is connected to an oscillator giving a fixed sinusoidal output voltage of 5.00 V r.m.s. with the oscillator set at a frequency of 50 Hz the r.m.s. current* *in the coil is 1.00 A and at a frequency of 100 Hz, the r.m.s. current is 0.625 A. (a) Determine the inductance of the coil (b) Calculate the ratio of the powers dissipated in the coil in the two cases.*

Solution:

In LR circuit,



Voltage (V) = 5.0V

Frequency (f_1) = 50 Hz

Current (I_1) = 1.0A

Frequency (f_2) = 100 HZ

Current (I_2) = 0.625 A

Impedance at frequency 50Hz is given by,

$$Z_1 = \frac{V}{I_1} = \frac{5}{1} = 5\,\Omega$$

Impedance at frequency 100 Hz is given by,

$$Z_2 = \frac{V}{I_2} = \frac{5}{0.625} = 8\,\Omega$$

Again, $Z_1^2 = R^2 + \omega_1^2 L^2$

or, $Z_1^2 = R^2 + 4\pi^2 f_1^2 L^2$

$Z_2^2 = R^2 + \omega_2^2 L^2$ .... (i)

Similarly, $Z_2^2 = R^2 + 4\pi^2 f_2^2 L^2$ .... (ii)

Subtracting eq. (ii) from (i)

$$Z_2^2 - Z_1^2 = (4\pi^2 f_2^2 - 4\pi^2 f_1^2) L^2$$

or, $$L^2 = \frac{Z_2^2 - Z_1^2}{4\pi^2 (f_2^2 - f_1^2)} = \frac{8^2 - 5^2}{4\pi^2[(100)^2 - (50)^2]}$$

$$= \frac{39}{4\pi^2 \times 7500} = \frac{39}{296088} = 0.000131717$$

$\therefore$ L = 0.0114 H

b) Power dissipated in the first case $P_1 = I_1^2 R$

Power dissipated in the 2nd case $P_2 = I_2^2 R$

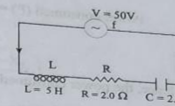
$$\therefore \frac{P_1}{P_2} = \frac{I_1^2 R}{I_2^2 R} = \left(\frac{1.00}{0.625}\right)^2 = 2.56$$

Example - 15. *An a.c. generator of constant 50 Vr.m.s. and a variable frequency is connected in series with a 2.0 Ω resistor, 5 H inductor and a 2.0 μF capacitor. The frequency is adjusted until the current in the circuit has a maximum value of 5 A r.m.s. Calculate the resistance of the wire of the inductor and the value of this frequency (Given π^2 = 10)*

Solution:

For the maximum current,

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{5 \times 2 \times 10^{-6}}}$$



$\therefore$ f = 50 Hz

Impedance of the circuit

$$Z = \frac{V}{I} = \frac{50}{5} = 10\,\Omega$$

But, $Z = \sqrt{(R+r)^2 + (X_L - X_C)^2}$

$Z = \sqrt{(R+r)^2} = R + r$ $[\because X_L = X_C]$

or, 10 = 2 + r

or, r = 8 Ω

Example - 16. *At what frequency would a 10 μF capacitor have reactance of 500 Ω?*

Solution:

Here,

Capacitance of capacitor (c) = 10 μF = 10×10^{-6} F

Reactance due to capacitor (X_c) = 500 Ω

We know, $X_C = \frac{1}{2\pi fc}$

or, $$500 = \frac{1}{2 \times \frac{22}{7} \times f \times 10 \times 10^{-6}}$$

or, $$500 = \frac{7 \times 10^6}{2 \times 22 \times f \times 10}$$

$$\therefore f = \frac{7 \times 10^6}{500 \times 2 \times 22 \times 10} = 31.8\text{ Hz}$$

Hence,

The frequency required = 31.8 HZ

Example - 17. *A series circuit has a resistance of 75 Ω and an impedance of 150 Ω. What power is consumed in the circuit when a p.d. of 120 V (r.m.s.) is impressed across it?*

Solution:

Here,

Resistance of circuit (R) = 75 Ω

Impedance (z) = 150 Ω

Potential difference applied (p.d) = 120V

Now, The current (I) = $\frac{V}{Z} = \frac{120}{150} = 0.8$ A

Now, Power consumed (P) = IV cosθ = $120 \times 0.8 \times \frac{R}{Z}$

$$= 120 \times 0.8 \times \frac{75}{150} = 48\text{W}$$

Hence, the power consumed in the circuit is 48 W.

Example - 18. *A coil of inductance 0.1 H and negligible resistance is in series with a resistance 40 Ω A supply voltage of 50 V (rms) is connected to them. If the voltage across L is equal to that across R, calculate the voltage across the inductor and frequency of the supply.*

Solution:

Here, inductance (L) = 0.1 H

Resistance (R) = 40 Ω

Supply voltage (V) = 50 V

Voltage across L = Voltage across R

$I.\,X_L = I.\,R$

$L\omega = R$

or, $2\pi fL = R$

$$\therefore f = \frac{R}{2\pi L} = \frac{40}{2\pi.\,0.1} = 63.66\text{ Hz}$$

Again, $Z = \sqrt{R^2 + X_L^2} = \sqrt{40^2 + (L2\pi f)^2} = 40\sqrt{2}$

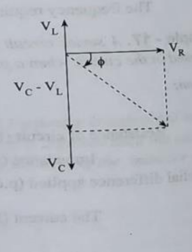
Then $I = \frac{V}{Z} = \frac{50}{40\sqrt{2}} = 0.884$ A

Hence, the voltage across the inductor (L) = $I.\,X_L$

$= 0.884 \times 2\pi \times 0.1 \times 63.66 = 35.4$ V

Example - 19. *A resistor of 100 Ω resistance and an inductor of 1 H inductance and a capacitor of 1 μF capacitance are placed in series with an a.c. source. Find the phase shift between current and voltage if the frequency of the a.c. supply is 50 Hz.*

Solution:



Here,

Resistance of circuit (R) = 100Ω

Inductance of inductor (L) = 1 H

Capacitance of capacitor (C) = 1 μF = 10^{-6} F

Frequency of the a.c supply = 50 Hz

Now,

Capacitive reactance (X_C) = $\frac{1}{2\pi fc}$

$$= \frac{1}{2\pi \times 50 \times 10^{-6}} = 3183.09\,\Omega$$

Inductive reactance (X_L) = $2\pi fL = 2\pi \times 50 \times 1 = 314.15\,\Omega$

Here, $X_C > X_L$

So, $X_C - X_L = 3183.09 - 314.15 = 2868.98\,\Omega$

Now, phase shift $\phi = \tan^{-1}\left(\frac{V_C - V_L}{V_R}\right) = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$

$$= \tan^{-1}\left(\frac{2868.93}{100}\right) = 88°$$

So, current leads voltage by 88°.

Example - 20. *Find the capacity of a capacitor which must be placed in series with a resistance of 50 ohm and inductance of 10 mH to bring the current in phase with the voltage, if the frequency of the a.c. supply be 50cycles/sec. What current will flow if 200 volts be impressed on the circuit?*

Solution:

Here, Resistance (R) = 50 Ω

Inductance (L) = 10 mH = 10×10^{-3} H

Frequency of a.c supply = 50 cycle/sec

Voltage applied (p. d) = 200 V

Now, Impedance (Z) = $\sqrt{R^2 + X_L^2} = \sqrt{R^2 + (2\pi fL)^2}$

$$= \sqrt{50^2 + \left(2 \times \frac{22}{7} \times 50 \times 10 \times 10^{-3}\right)^2} = 50.098\,\Omega$$

Now, $I = \frac{v}{z} = \frac{200}{50.098} = 4$A

Since, the frequency of a.c supply is same so,

$X_L = X_C$

or, $2\pi fL = \frac{1}{2\pi fc}$

or, $4 \times \frac{22^2}{7^2} \times 50^2 \times 10 \times 10^{-3} = \frac{1}{c}$

$\therefore$ c = 0.001 F

Capacitance of the capacitor is 0.001 F.

Example - 21. *A series circuit consists of a resistor of 50 Ω, an inductor of 10 mH and a capacitor of 10 μF. It is connected across a 200 V, 50 Hz line. What would a voltmeter read when it is connected across the resistor?*

Here,

Resistance (R) = 50 Ω

So, Inductive reactance $(X_L) = 2\pi f \times L = 2 \times \frac{22}{7} \times 50 \times 10 \times 10^{-3} = 3.1428\ \Omega$

Capacitance (c) = $10\mu F = 10 \times 10^{-6}$ F

So, capacitive reactance $(x_c) = \frac{1}{2\pi fc} = \frac{7\times10^6}{2\times22\times50\times10} = 318.18\ \Omega$

Now, Impedance (z) = $\sqrt{R^2+(X_C-X_L)^2}$

$= \sqrt{50^2+(318.18-3.1428)^2} = 318.98\Omega$

So, $I = \frac{V}{Z} = \frac{200}{318.98} = 0.626$ A

Now, voltmeter reading across Resistance

$V = IR = 50 \times 0.626 = 31.3$ V

Example - 22. *A resistor 5 Ω is connected in series with an inductor of 22 mH and a capacitor of 100 μF and a.c. supply of 260 V, 1000 Hz (a) What is the power factor of the circuit (b) What is the power dissipated (c) What is the resonant frequency of the circuit.*

Solution:

Here,

Voltage (V) = 260 V

Frequency (f) = 1000 Hz

Resistance of resistor (R) = 5Ω

Inductance (L) = 22 mH = 22×10^{-3} H

Capacitance (C) = 100 μF = 100×10^{-6} F

Now, Inductive reactance $(X_L) = 2\pi f \times L = 2 \times \frac{22}{7} \times 1000 \times 22 \times 10^{-3}$

$= 138.2857\ \Omega$

Capacitance reactance $(X_c) = \frac{1}{2\pi fc} = \frac{7\times10^6}{2\times22\times1000\times100}$

$= 1.5909\ \Omega$

∴ Impedance (z) = $\sqrt{R^2+(X_L-X_C)^2} = \sqrt{9^2+(138.285-1.5o9)^2}$

Now, Current (I) = $\frac{V}{z} = \frac{260}{137.05} = 1.9007$ A

Power consumed in circuit = IV cosϕ

Now, $\cos\phi = \frac{R}{Z}$

or, $\cos\phi = \frac{5}{137.05} = 0.037$

So, Power dissipated = I Vcosϕ

$= 1.9007 \times 260 \times 0.037$

$= 18.1$ W

For LCR circuit

Resonant frequency (f) = $\frac{1}{2\pi\sqrt{Lc}} = \frac{1}{2\times\frac{22}{7}\times\sqrt{22\times10^{-3}\times100\times10^{-6}}}$

$= 107.3$ Hz

Example - 23. *A 110 V, 100 W lamp is to be run at 220 V, 53 cycle mains. Calculate the inductance of the choke to be placed in series with the lamp?*

Solution:

Here,

Voltage of the lamp (V) = 110 V

Power rated of the bulb (P) = 100W

So, $P = \frac{V^2}{R}$

or, $R = \frac{V^2}{P} = \frac{110^2}{100} = 121\ \Omega$

Also, V = IR

or, $110 = I \times R$

∴ Current in the circuit (I) = 0.90909 A

Since, the current to be used by the lamp remains the same, so,

$I = \frac{E_v}{Z}$

Where, E_V = Voltage across the combination of Resistance & Inductance

z = Impedance

∴ z = 242 Ω

Also, $z = \sqrt{R^2+X_L^2}$

or, $(242)^2 = 121^2 + X_L^2$

or, $X_L^2 = 242^2 - 121^2$

or, $X_L^2 = 43923$

or, $4 \times \left(\frac{22}{7}\right)^2 \times 53^2 \times L^2 = 43923$

$L^2 = \frac{43923}{4\times\left(\frac{22}{7}\right)^2\times53^2}$

∴ L = 0.63 H

Example - 24. *A coil having inductance 10 mH and resistance 10 Ω is connected across a 220 V, 50 Hz line. Compute (i) the current in the coil (ii) the phase angle between the current and the supply voltage (iii) the power loss in the coil.*

Solution:

Here,

Inductance of coil (L) = 10 mH = 10×10^{-3} H

Resistance (R) = 10 Ω

Inductive reactance $(X_L) = 2\pi fL = 2 \times \frac{22}{7} \times 50 \times 10 \times 10^{-3}$

$= 3.143\ \Omega$

∴ Impedence of circuit (Z) = $\sqrt{R^2+X_L^2} = \sqrt{10^2+(3.143)^2}$

$= 10.482\Omega$

Hence, Current in the coil (I) = $\frac{V}{Z} = \frac{220}{10.482} = 20.99$ A

For the phase angle between the current & voltage supply

$\tan\phi = \frac{X_L}{R} = \frac{3.143}{10}$

∴ ϕ = 17.4°

Power loss in the coil = VI cosϕ = $220 \times 21 \times \frac{R}{Z} = 220 \times 21 \times \frac{10}{10.482}$

$= 4407.55581 = 4407.56$ W

Example - 25. *An alternating voltage of r.m.s. value, 283 V, 50 Hz is applied to a coil of 20 Ω resistance and 10 mH inductance. Find the r.m.s. current and angle by which the current lags behind the voltage.*

Solution:

Here,

Voltage of a.c. supply (V) = 283 V

Frequency of a.c supply (f) = 50 Hz

Resistance of coil (R) = 20 Ω

Inductance (L) = 10 mH = 10×10^{-3} H

∴ Inductive reactance $(X_L) = L\omega = 2\pi fL = 2 \times \frac{22}{7} \times 50 \times 10 \times 10^{-3}$

$= 3.143\ \Omega$

Now, Impedence of circuit (z) = $\sqrt{R^2+X_L^2}$

$= \sqrt{20^2+3.143^2} = 20.245\ \Omega$

So, R.M.S. current $(I_{rms}) = \frac{V_{rms}}{z} = \frac{283}{20.245} = 13.97$A

Also, $\tan\phi = \frac{X_L}{R}$

or, $\tan\phi = \frac{3.143}{20}$

∴ ϕ = 8.9°

Hence, the current lags behind the voltage by (8.9°).

Example - 26. *A resistor of 50 Ω, an inductor of 10 mH and a capacitor of 1μF is connected to an a.c. source of variable frequency. At what frequency the current will be maximum in the circuit? What is the value of maximum current if the supply voltage is 12 V?*

Solution:

Resistance of resistance (R) = 50 Ω

Inductance of inductor (L) = 10 mH = 10×10^{-3} H

Capacitance of capacitor (C) = 1μF = 1×10^{-6} F

For the maximum value of frequency of a.c supply we know

$f = \frac{1}{2\pi\sqrt{LC}} \frac{1}{2\times\frac{22}{7}\sqrt{10\times10^{-3}\times1\times10^{-6}}} = 1591$ Hz

Now, inductive reactance $(X_L) = 2\pi fL = 2 \times \frac{22}{7} \times 1591 \times 10 \times 10^{-3} = 100\ \Omega$

Capacitive reactance $(X_C) = \frac{1}{2\pi fc} = \frac{7\times10^6}{2\times22\times1591\times1} = 99.994\ \Omega$

Hence, Impedence of circuit (z) = $\sqrt{R^2+(X_L-X_C)^2} = \sqrt{50^2+(100-99.994)^2}$

$= 50.00006$

$z \approx 50\ \Omega$

Now, Value of maximum current supply (I) = $\frac{V}{Z} = \frac{12}{50} = 0.24$ A

Example - 27. *A resistor of 48 Ω and an inductor of $\frac{7}{50\pi}$ H are connected to an a.c. source of 200 V, 50 Hz. Calculate (a) the peak voltage (b) the r.m.s. current (c) the r.m.s. voltage across the coil.*

Solution:

Here,

Resistance of resistor (R) = 48 Ω

Inductance of inductor (L) = $\frac{7}{50\pi}$ H

So, Inductive reactance $(X_L) = 2\pi fL = 2\pi 50 \times \frac{7}{50\pi} = 14\Omega$

Impedence of circuit (z) = $\sqrt{R^2 + X_L^2} = \sqrt{48^2 + 14^2} = 50\Omega$

The peak voltage of a circuit is given by

$$V_o = \sqrt{2}\,V = \sqrt{2} \times 200 = 283 \text{ V}$$

Now, peak current of the circuit is given by

$$I_o = \frac{V_o}{R} = \frac{283}{50} = 5.66 \text{ A}$$

But, the r.m.s. current $(I_V) = 0.707 \times I_o = 0.707 \times 5.66 = 4\text{A}$

The r.m.s voltage across the coil of inductive reactance $(V_1) = I_V X_L$

$= 4 \times 14 = 56$ V

Example - 28. *An alternating e.m.f. of 6.0 V (r.m.s.) and frequency 50 Hz is applied to a circuit consisting a capacitor of 2 μF and a resistor of 500.0 Ω in series (i) Find the current flowing (ii) the voltage across the capacitor (iii) the phase angle between the applied e.m.f. and current (iv) the average power supplied.*

Solution:

Here,

R. M. S voltage of a.c supply (Vrms) = 6.0 V

Frequency of an a.c supply (f) = 50 Hz

Capacitance of capacitor (c) = 2μF = 2×10^{-6} F

$\therefore$ Capacitance reactance $(x_c) = \frac{1}{2\pi fc} = \frac{7\times10^6}{2\times22\times50\times2} = 1590.909\ \Omega$

Resistance (R) = 500 Ω

$\therefore$ Impedance (z) = $\sqrt{X_c^2 + R^2} = \sqrt{1590.909^2 + 500^2}$

$= 1667.63\ \Omega$

So, the current flowing the circuit (I) = $\frac{V}{Z} = \frac{6}{1667.63} = 3.6 \times 10^{-3}$ A

$= 3.6$ mA

Also, the voltage across the capacitor $(V_1) = I \times X_C = 3.6 \times 10^{-3} \times 1590.909$

$= 5.7$ V

The phase angle between the applied e.m.f. and current

$\tan\phi = \frac{X_C}{R} = \frac{1590.909}{500}$

$\therefore \tan\phi = 3.1818$

$\therefore \phi = 72.6°$

So, the average power supplied = IV cosϕ

$= 3.6 \times 10^{-3} \times 6 \times \cos 72.6$

$= 6.47 \times 10^{-3}$ W

$= 6.47$ mW

Example - 29. *A 100 Ω resistor, 1μF capacitor and a 0.1 H inductor are in series with an a.c. voltage of 6.0 V (r.m.s.). Its frequency is kept constant and it frequency is varied from a low to a high value. Calculate the resonating frequency, the maximum current and the voltage across the capacitor at resonance.*

Solution:

Here, in LR circuit,

Inductance of inductor (L) = 0.1 H

Capacitance of capacitor (C) = $1\mu F = 10^{-6}$ F

Resistance (R) = 100Ω

(a) If f denotes the resonate frequency then

$$f = \frac{1}{2\pi\sqrt{Lc}} = \frac{7}{2\times22\times\sqrt{0.1\times10^{-6}}} = 503 \text{ Hz}$$

Now, inductive reactance $(X_L) = 2\pi fL = 2 \times \frac{22}{7} \times 503 \times 0.1 = 316.17\Omega$

Capacitive reactance $(X_C) = \frac{1}{2\pi fc} = \frac{7\times10^6}{2\times22\times503} = 316.28\ \Omega$

$\therefore$ Impedance of the circuit (z) = $\sqrt{R^2 + (X_L - X_C)^2}$

$= \sqrt{100^2 + (316.17 - 316.28)^2} = 100.36\ \Omega$

(b) $\therefore$ The maximum current (I) = $\frac{V}{Z} = \frac{6}{100.36} = 0.06$A

(c) The voltage across the capacitor $(V_1) = IX_C = 0.06 \times 316.28 = 18.9$ V

Example - 30. *An inductance coil having a reactance of 25 Ω gives off heat at the rate of 10.0 Js^{-1} wh[en] it carries a current of 0.5 A (r.m.s.), what is the impedance of the coil.*

Solution:

Here,

Inductive reactance of the coil $(X_L) = 25\ \Omega$

Power consumed or heat produced per sec (P) = 10.0 W

Current supplied $(I_{rms}) = 0.5$ A

We know, P = VI cosϕ

$P = V_{rms}\, I_{rms} \cos\phi$

or, $P = I_{rms} \times z \times I_{rms} \times \frac{R}{Z}$

or, $P = I_{rms}^2 \times R$

or, $10.0 = 0.5^2 \times R$

$\therefore$ R = 40 Ω

Now, Impedecnce of the circuit (Z) = $\sqrt{R^2 + X_L^2} = \sqrt{40^2 + 25^2} = 47.16\ \Omega$

Example - 31. *An inductor, a resistor and a capacitor are connected in series across an a-c. circuit A voltmeter reads 60 V when connected across the inductor, 16 V across the resistor and 30 V across the capacitor (i) What will the voltmeter reading when placed across the series circuit (ii) What is the power factor of the circuit ?*

Solution:

Here,

Inductor, capacitor and resistor are connected in series across an a.c. circuit

Across the inductor

Reading of Voltmeter $(V_L) = 60$V

We know, $V = IX_L$

or, $60 = I \times X_L$ where X_L is the inductive reactance

$\therefore X_L = \frac{60}{I}$

Across the resistor

Reading of voltmeter $(V_R) = 16$ V

We know, V = IR

or, 16 = IR

$\therefore R = \frac{16}{I}$

Across the capacitor

Reading of voltmeter $(V_C) = 30$V

We know, $V = IR$ Where X_C is the capacitive reactance

or, $30 = I X_C$

$\therefore X_C = \frac{30}{I}$

Impedence of circuit (z) = $\sqrt{R^2 + (X_L - X_C)^2} = \sqrt{\left(\frac{16}{I}\right)^2 + \left(\frac{60}{I} - \frac{30}{I}\right)^2}$

$= \sqrt{\frac{1}{I^2}\left\{16^2 + (60-30)^2\right\}} = \frac{1}{I}\sqrt{256 + 900} = \frac{34}{I}$

Now, the voltage across the a.c. circuit (V) = IZ

or, $V = I \times \frac{34}{I}$

$\therefore$ V = 34 V

Power factor of the circuit $(\cos\phi) = \frac{R}{Z} = \frac{\frac{16}{I}}{\frac{34}{I}} = 0.47$

Example - 32. *A radio can tune over the frequency of a portion of MW broadcast band (880 KHz to 1500 KHz). If its L.C. circuit has an effective inductance of 400 μH, what would be the range of its variable capacitor?*

Solution:

Here,

Effective inductance (L) = 200 μH = 200×10^{-6} H

Frequency of broadcast band (f) = 880 KHZ to 1500 KHz

For, 880 KHZ

Effective frequency (f) = $\frac{1}{2\pi\sqrt{LC}}$

$$880 \times 1000 = \frac{7}{2\times22\sqrt{400\times10^{-6}\times C_1}}$$

$$C_1 = \frac{49\times10^6}{2^2\times22^2\times880^2\times1000^2\times400} = 8.17 \times 10^{-11} \text{ F}$$

For 1200 KHZ

Effective frequency (f) = $\frac{1}{2\pi\sqrt{LC}}$

$$1200 \times 1000 = \frac{1}{2\times\frac{22}{7}\times\sqrt{400\times10^{-6}\times C_2}}$$

$$C_2 = \frac{49\times10^6}{4\times22^2\times1200^2\times1000^2\times400}$$

$= 4.394 \times 10^{-11}$ F

Hence, the range of its variable capacitor will be

4.394×10^{-11} F to 8.17×10^{-11} F.