PROJECTILE MOTION

- Motion of particle in a plane which is thrown into space and moves under the influence of gravity alone with constant acceleration is called projectile motion.

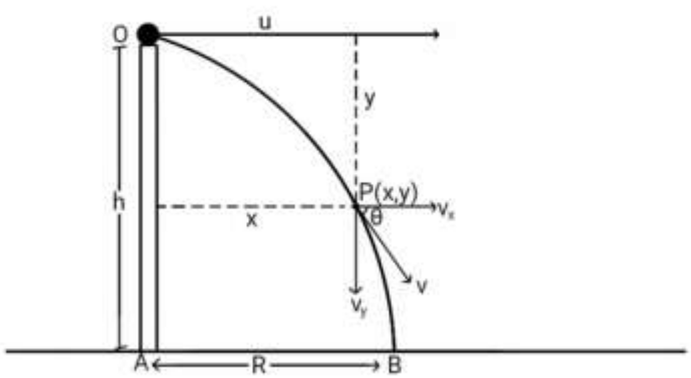
- In projectile motion, acceleration due to gravity is constant and air resistance is negligible.

- The path of a projectile motion is parabolic.

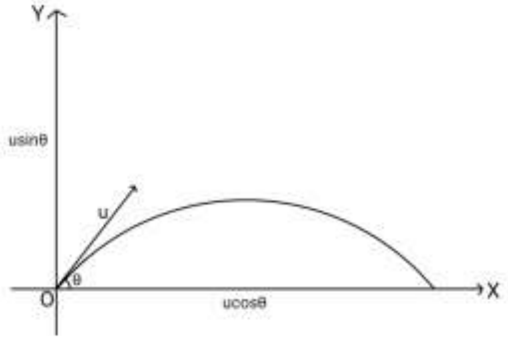
Projectile motion

Physical Quantity during projectile motion	**Horizontal projection from a height**	**Angular projection of projectile from ground making angle 'θ' with horizontal**
Velocity of projectile at any instant 't'	$v = \sqrt{u^2 + g^2t^2}$	$v = \sqrt{u^2 + g^2t^2 - 2ugt\sin\theta}$
Direction of velocity with horizontal	$\beta = \tan^{-1}\left(\frac{gt}{u}\right)$	$\beta = \tan^{-1}\left(\frac{u\sin\theta - gt}{u\cos\theta}\right)$
Time of flight	$T = \sqrt{\frac{2h}{g}}$	$T = \frac{2u\sin\theta}{g}$
Horizontal range	$R = u\sqrt{\frac{2h}{g}}$	$R = \frac{u^2\sin 2\theta}{g}$
Maximum height	$H = h$	$H = \frac{u^2\sin^2\theta}{2g}$

Parabolic path of projectile when projected horizontally from a height



Parabolic path of projectile when projected horizontally from ground



Important facts of angular projection of projectiles

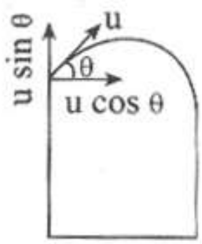
- For maximum horizontal range
  $\sin 2\theta = 1$
  $\theta = 45$
  $R_{max} = \frac{u^2}{g}$
- For the least possible speed to attain a certain range, angle of projection should be $45^o \left(u^2 \propto \frac{1}{\sin\ 2\theta}\right)$ and the least speed is $u_{min} = \sqrt{gR}$
- If the angles of projection are θ and $90^o - \theta$ then
  (a) Horizontal range remains the same i.e, $R_1 : R_2 = 1 : 1$
  (b) $H_1 : H_2 = \tan^2\theta : 1$
  (c) $T_1 : T_2 = \tan\theta : 1$
  (d) $R_1 = R_2 = R = 4\sqrt{H_1 H_2}$
  (e) $H_1 + H_2 = \frac{v^2}{2g}$

- If a person can throw to a maximum horizontal distance 'R', then the maximum height up to which he can throw is $\frac{R}{2}$.
- If 'k' be the initial K.E. of body, then K.E. at highest point $\frac{1}{2}mu^2\cos^2\theta = k\cos^2\theta$
  P.E. at highest point = $mHg = \frac{mg.u^2\sin^2\theta}{2g} = k\sin^2\theta$

- If range is 'n' times the maximum height, then $\theta = \tan^{-1}\left(\frac{4}{n}\right)$.
  {Comes from the formula : $R\tan\theta = 4H$ (easy way to remember : Ratan Chor Ho)}

- If the equation of trajectory is $y = ax - bx^2$, then
  (a) Angle of projection (θ) = $\tan^{-1}a$
  (b) Horizontal range (R) = $\frac{a}{b}$
  (c) Maximum height (H) = $\frac{a^2}{4b}$

➲ **Projectile at an angle 'θ' above horizontal**



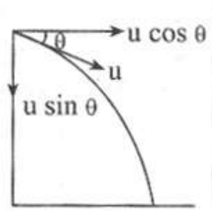
For horizontal motion

x = (u cos θ). t

For vertical motion

$h = (-u \sin \theta)\, t + \frac{1}{2} g t^2$

➲ **Projectile at an angle 'θ' below horizontal**



For horizontal motion

x = (u cos θ). t

For vertical motion

$h = (u \sin \theta)\, t + \frac{1}{2} g t^2$

Projection from a moving body

(1) Ball is projected in direction of the motion of a vehicle
Horizontal component of ball's velocity = u cos θ + v
Vertical component of ball's velocity = u sin θ

(2) Ball is projected in the direction opposite to the motion of a vehicle
Horizontal component = u cos θ – v
Vertical component = u sin θ

(3) Ball projected from an upward-moving platform
Horizontal component = u sin θ
Vertical component = u sin θ + v

(4) Ball projected from a downward-moving platform
Horizontal component = u cos θ
Vertical component = u sinθ – v

Where,
u = velocity of ball
v = velocity of vehicle or platform