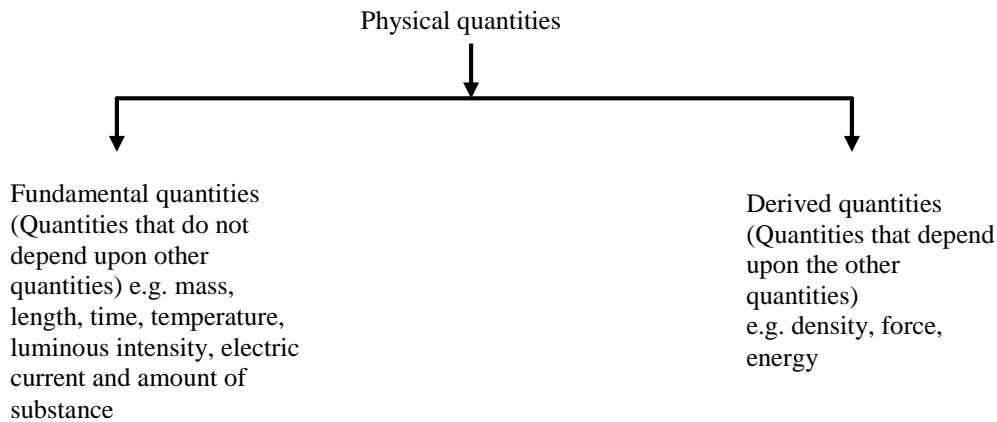


Units, Dimensions and Errors

- The measurable quantities that describe the laws of physics are called physical quantities.



- Units of plane angle (radian) and solid angle (steradian) are called supplementary units.
- The product of numerical value (n) and unit (u) always remains constant for a measurement i.e.,

$$nu = \text{constant}$$

$$n \propto \frac{1}{u}$$

- Dimensions of a physical quantity are the powers to which the fundamental units are raised to obtain one unit of that quantity.
- A physical quantity has unique dimensions but a particular dimension does not represent a unique physical quantity.
- A numerically correct physical relation is also dimensionally correct but a dimensionally correct relation may not be numerically correct.
- Some important units:
 - 1 x-ray unit = $10^{-13}\text{m} = 0.001\text{\AA}$
 - 1 astronomical unit (AU) = $1.5 \times 10^{11}\text{m}$ (mean distance between earth and sun)
 - 1 mile = 1.61 km
 - 1 shake = 10^{-8}sec .
 - 1 bar = 1 atmosphere = $1.01 \times 10^5 \text{ Pa}$.
 - 1 torr = 133.3 Pa
 - 1 erg = 10^{-7} J
 - 1 barn = 10^{-28} m^2
 - 1 gauss = 10^{-4} T
 - 1 oersted = 79.58 Am^2
 - 1 fermi = 1 fm = 10^{-15} m
 - 1 light year = distance travelled by light in vacuum in 1 year = $9.46 \times 10^{15} \text{ m}$
 - 1 electron volt = $1.6 \times 10^{-19} \text{ J}$

- Dimensional Formula of Some Quantities and Constants

- Wien's constant (b) = $\lambda_m \times T \rightarrow [M^0 L T^0 K]$
- Stefan's constant (σ) = $\frac{E}{T^4} \rightarrow [ML^0 T^{-3} K^{-4}]$
- Permittivity of Free Space (ϵ_0) = $\frac{q_1 q_2}{4\pi F r^2} [M^{-1} L^{-3} T^4 A^2]$
- Self-Inductance (L) = $\epsilon \frac{dt}{di} = [ML^2 T^{-2} A^{-2}]$
- Magnetic Field Intensity (B) = $\frac{F}{qv} = [MT^{-2} A^{-1}]$
- Magnetic Moment (M) = $IA = [M^0 L^2 T^0 A^1]$
- Capacitance (C) = $\frac{q}{V} = [M^{-1} L^{-2} T^4 A^2]$
- Rate of flow (v) = $\frac{Vol.}{t} = [M^0 L^3 T^{-1}]$
- Velocity gradient = $\frac{Vel.}{distance} = [M^0 L^0 T^{-1}]$
- Relative velocity = velocity = $[M^0 L^1 T^{-1}]$

➤ Some Dimensionless Quantities

- Plane Angle/Phase
- Solid Angle
- Reflectance, Absorptance and Transmittance
- Strain
- Relative Density
- Relative permeability
- Magnetic susceptibility
- Relative Permittivity
- Emissivity
- Refractive Index
- Poisson's Ratio
- Power Factor

➤ Quantities having same dimensional units :

	Quantities	Dimensions
1.	Frequency, angular frequency, angular velocity, velocity gradient	$[M^0 L^0 T^{-1}]$
2.	Work, Energy, Torque, Moment of Force	$[M^1 L^2 T^{-2}]$
3.	Pressure, stress, Young's modulus, Bulk modulus, modulus of rigidity, energy density	$[ML^{-1} T^{-2}]$
4.	Momentum, impulse	$[M^1 L^1 T^{-1}]$

5.	Thrust, force, weight, energy gradient	[MLT ⁻²]
6.	Surface tension, surface energy, force gradient, spring constant	[M ¹ L ⁰ T ⁻²]
7.	Angular momentum and Planck's constant	[M ¹ L ² T ⁻¹]
9.	If C, R and L are capacitance, resistance and inductance, then RC, $\frac{L}{R}$, \sqrt{LC} have same dimensions.	[M ⁰ L ⁰ T]

➤ **Homogeneity Principle of Dimensional Analysis**

Principle of Homogeneity states that dimensions of each of the terms of a dimensional equation on both sides should be the same. This principle is helpful because it helps us convert the units from one form to another.

➤ **Error and measurement:**

If a_t be the true value of physical quantity and a_m be the measured value then,

(a) Absolute error $\Delta a_i = |a_t - a_m|$

(b) Relative or fractional error = $\frac{\text{absolute error}}{\text{true value}}$

(c) Percentage error = relative error $\times 100\%$

$$= \frac{\Delta a_i}{a_t} \times 100\%$$

➤ **Calculation of errors:**

S.N.	Mathematical operation	Error
1.	Addition $x = a + b$	$\Delta x = \pm (\Delta a + \Delta b)$
2.	Subtraction $x = a - b$	$\Delta x = \pm (\Delta a + \Delta b)$
3.	Multiplication $x = a \times b$	$\frac{\Delta x}{x} = \pm \left(\frac{\Delta a}{a} + \frac{\Delta b}{b} \right)$
4.	Division $x = \frac{a}{b}$	$\frac{\Delta x}{x} = \pm \left(\frac{\Delta a}{a} + \frac{\Delta b}{b} \right)$
5.	Exponential $x = a^n$	$\frac{\Delta x}{x} = \pm n \left(\frac{\Delta a}{a} \right)$
6.	Power of $x = a^n b^m / c^p$	$\frac{\Delta x}{x} = \pm \left(n \frac{\Delta a}{a} + m \frac{\Delta b}{b} + p \frac{\Delta c}{c} \right)$

➤ Error in any instrument is equal to the least count of that instrument.